PRIMARY PRODUCTION AND ECONOMIC DEVELOPMENT

BY
MUNIDASA CHANDRASOMA KODIKARA

TECHNICAL REPORT NO. 98
NOVEMBER 28, 1961

PREPARED UNDER CONTRACT Nonr-225(50)
(NR-047-004)
FOR
OFFICE OF NAVAL RESEARCH

INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
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CHAPTER I

THE PROBLEM AND ITS SETTING

1.1 Introduction.

The prices, output and the proceeds from the production of primary commodities have been in the past subject to rather violent fluctuations. The instability arising from these fluctuations is not only a source of instability to the producer countries, but is also a case for concern to the industrial countries who use these commodities.

Underdeveloped countries are not the only producers of primary commodities. However, the problems of primary producing underdeveloped countries are more serious. Their economic backwardness and the weaker bargaining strength make them especially vulnerable to fluctuations.

The problems of primary producing underdeveloped countries has been the subject of many recent research studies. The United Nations and its specialized agencies have sponsored many such studies.[1] A resolution of the General Assembly of the United Nations[2] in 1952 resulted in the appointment of a committee of five economists to study the excessive fluctuations in the primary commodity markets, and to suggest practical

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[1] Two such studies that we shall refer to in the future are; United Nations, Department of Economic Affairs ; Instability in Export markets of Under-developed Countries, (New York 1952); and United Nations, Commodity Trade and Economic Development, (New York 1953).

[2] Resolution 623 (VII), of the General Assembly of the United Nations, adopted 21 December 1952; this resolution, the terms of reference of the committee and the report of the committee, are published in Commodity Trade and Economic Development, op. cit.

methods for their stabilization. More recently, the contracting parties of the General Agreement on Tariffs and Trade (GATT) appointed a panel consisting of four of the world's foremost authorities in international economics to study, among other things, the effects of the wide fluctuations of primary products and the export earnings of these countries.[3] Still more recently Kyklos, the international review of social sciences, conducted two symposiums devoted to the study of this same problem. The first of these symposiums,[4] consisting of an entire issue of this journal, was a debate between Professor Nurkse and a group of other well-known economists about the method of stabilization of the primary commodity markets, which was suggested by Professor Nurkse. The second[5] conducted about a year later was devoted to the more important 'dynamic' problem of economic development and stabilization. All these studies point to the seriousness of the problem of instability arising from the fluctuations in primary commodities and the very urgent need for a solution to this problem.

1.2 Factual background.

Before proceeding to discuss the problem resource allocation in a primary producing country, we intend taking a look at the nature of the fluctuations that arise in primary production. The Department of Economic Affairs of the United Nations, in their study entitled "Instability in Export Markets of Underdeveloped Countries,"[6]

3/ General Agreement on Tariffs and Trade, Trends in International Trade: a report by a panel of experts, (Geneva, 1958).

4/ "The Quest for a Stabilisation Policy in Primary Producing Countries," A symposium, Kyklos, Vol. 11, 1958, Fasc. 2, pp. 139-265.

5/ "Stabilisation and Development of Primary Producing Countries," Symposium 2, Kyklos, Vol. 12, 1959, Fasc. 3, pp. 269-401.

6/ Instability in Export Markets of Under-developed Countries, op. cit.

has collected and analyzed the fluctuations of prices and volumes of major primary commodities and the fluctuations in export earning of primary producing underdeveloped countries. The source of our statistics for this section is this excellent study.

First we shall consider the year-to-year and within-year fluctuations of the market prices of some of the primary commodities. The average percentages of these fluctuations for 20 major commodities are represented in Table 1.1. The year-to-year price fluctuations are measured as the change from the average of one year to the average of the next, measured as a percentage of the higher value. These prices, we see, are subject to very substantial fluctuations, varying from 7.1% for tea to 20.6% for rubber, with an average of 14.56%. This means that the prices of the next year could drop by 14.5% or increase by 17.0%.7/ The high value for this can be seen by comparison to the year-to-year variation of the unit value of the United States exports of finished manufactured goods; for the inter-war period, this was 6.4%,8/ less than half the average year-to-year fluctuation of the primary commodities. The year-to-year variation of the United States consumer price index was only 4%.9/

Within-year variation of the eleven price commodities show much larger fluctuations. The range of within-year variation is measured

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7/ This is the percentage increase from 100, as measured from the lower value 100.

8/ Commodity Trade and Economic Development, op. cit., p. 8.

9/ Ibid., p. 9.

TABLE 1.1

YEAR-TO-YEAR AND WITHIN-YEAR FLUCTUATIONS IN
MARKET PRICES OF TWENTY PRIMARY COMMODITIES 1901-1951
(Average Percentage Fluctuations)

Commodity	Year-to-Year[a]	Within Year[b]
Bananas	13.0	-
Cocoa	16.7	31.9
Coffee	17.2	26.2
Copper	13.8	21.9
Copra	16.1	-
Cotton	15.4	30.4
Hemp	16.1	-
Jute	15.0	33.3
Linseed	12.7	-
Manganese	18.7	-
Rice	12.8	-
Rubber	20.6	37.3
Silk	13.8	-
Sodium Nitrate	7.3	-
Sugar	17.1	18.7
Tea	7.1	29.2
Tin	13.3	26.0
Tobacco	18.4	-
Wheat	11.9	30.0
Wool	14.2	27.2
Average	14.37	28.37

Source: United Nations, Department of Economic Affairs, Instability in Export Markets of Underdeveloped Countries, (New York, 1952), pp. 10 and 22.

[a]Change from the average of one year to the average of the next, measured as a percentage of the higher value.

[b]The difference between the highest and lowest monthly prices within a year, measured as a percentage of the higher value.

as the difference between the highest and lowest monthly prices within a year, measured as a percentage of the higher value. In almost all cases, the within-year variation is much higher than the year-by-year variations. The within-year variations have an average of 28.4%, almost twice as high as the average year-to-year fluctuations. These wide short-run fluctuations are the combined result of secular, cyclical, yearly, seasonal, and random forces. The seasonal factor, very likely, is quite significant for many of the primary commodities.

The fact that the prices tend to move in the same direction for several successive years makes the measurement of year-to-year fluctuations alone underestimate degree of instability caused by price movements. We give in Table 2 an analysis of the cyclical movement of prices for the above 20 commodities, as compiled by the United Nations study. In the analysis of cyclical movement of prices, the long-term secular trends are eliminated. The average duration of the upswing and downswing of these price cycles is a little over two years, making a cycle of length a little over four years. The amplitude of a cycle, from trough to peak or from peak to trough, is about 29%, measured as a percentage of the higher value. This illustrates the high intensity of the price movements. For coffee, cotton, sugar, rubber, and tin, the amplitude was over 33-1/3%, measured as a percentage of the higher value for this makes price fluctuations of greater than 50% measured from the known value. The average rate of increase per year, resulting from the upswing of the cycle was 12.75%, and the rate of decrease during the downswing was about 14.4%. These annual rates of change due to cyclical movements are as high as the

TABLE 1.2

CYCLICAL MOVEMENT IN MARKET PRICES, 1901 to 1951

	Commodities	Upswing: Number of Up Swings	Upswing: Total Duration (Years)	Upswing, Percentage Increase: Total Amplitude[a]	Upswing, Percentage Increase: Rate Per Year	Downswing: Number of Downswings	Downswing: Total Duration (Years)	Downswing, Percentage Decrease: Total Amplitude[a]	Downswing, Percentage Decrease: Rate Per Year
1	Bananas	6	13	30	12	5	13	24	11
2	Cocoa	8	20	34	15	8	18	36	16
3	Coffee	7	21	43	15	8	19	42	18
4	Copper	7	22	36	12	7	19	37	14
5	Copra	4	9	30	14	4	11	32	18
6	Cotton	6	19	41	13	7	20	41	14
7	Hemp	12	20	22	12	12	19	23	15
8	Jute	9	19	30	16	9	20	31	14
9	Linseed	8	26	30	10	8	15	29	15
10	Manganese	5	10	27	12	4	8	36	29
11	Rice	10	21	24	13	10	21	24	12
12	Rubber	11	21	35	24	11	20	32	18
13	Silk	10	19	24	14	10	19	25	13
14	Sodium-Nitrate	10	19	12	5	10	23	13	6
15	Sugar	8	20	37	16	8	19	39	16
16	Tea	8	17	13	6	8	24	14	5
17	Tin	6	19	36	13	6	23	36	9
18	Tobacco	8	17	19	10	8	20	21	18
19	Wheat	10	22	27	10	10	21	17	8
20	Wool	9	21	29	13	9	21	31	17
	Average	8.1	18.75	28.95	12.75	8.15	18.65	29.15	14.3
	Whole sale price index:								
	United States	9	22	13.7	5.2	10	16	12.0	7.6
	United Kingdom	7	27	19.2	5.8	7	14	18.6	9.2

Source: Instability in Expert Markets of Underdeveloped Countries, op. cit. p. 17.

[a]Increase from the lower point measured as a percentage of the higher point.

average year-to-year variation of 14.56%; which means that much of the year-to-year price variations arise from the cyclical movements in prices. For a comparison of the amplitude of these cycles, we give at the end of Table 1.2 an analysis of the cycles of wholesale price indexes of the United States and the United Kingdom. We see that the amplitude of these cycles is about half those of primary commodities, and so is the rate of change per year. Therefore, we see that the price fluctuations in primary commodities are twice as high as variations in the normal price cycle.

Prices are not the only source of instability of primary producing countries. The volume of output of these products is also subject to large variations. In fact, their variation is greater than that of prices. In Table 1.3, we give the year-to-year fluctuations in the export volume of ten important primary commodities, broken down by major producer countries of the products. The extent of annual change is given, as before, as a percentage of the higher value. The average for each commodity has been calculated by weighting according to the exports of the underdeveloped country. For the ten commodities, we see that with the exception of coffee and sugar, the fluctuations in volume were greater than the year-to-year fluctuations in price. The average fluctuation in volume was around 18% while that of prices was around 14.5%, so that it may be said that on the average the fluctuations in volume were greater than those of price.

Another characteristic of the volume variations is the great diversity between different countries producing the same product. For instance, in the case of cotton, fluctuation in the export volume of Brazil was 46.0%, as compared to 14.2% of Egypt.

TABLE 1.3

YEAR-TO-YEAR FLUCTUATIONS IN EXPORT VOLUME

(Average Percentage Fluctuations Per Year)

Commodity and Country:	Period Covered	Year-to-Year Fluctuations[a]
Cocoa:		
Brazil	1910-1950	16.0
Gold Coast (Ghana)	1910-1950	16.2
Trinidad and Tobago	1910-1950	22.8
Average		16.7
Coffee:		
Brazil	1902-1950	16.4
Other Countries	1902-1950	8.3
Average		12.4
Copper:		
Chile	1902-1950	16.1
Mexico	1902-1950	18.5
Peru	1902-1950	11.9
Average		16.3
Cotton:		
Brazil	1904-1950	46.0
China	1904-1944	28.9
Egypt	1904-1950	14.2
India and Pakistan	1904-1950	22.5
Peru	1905-1950	20.1
Average		21.3
Rubber:		
Indonesia	1911-1950	28.6
Malaya	1906-1946	29.4
Average		29.0
Sodium Nitrate:		
Chile	1914-1950	22.2
Tea:		
Ceylon	1910-1950	5.6
China	1910-1950	21.5
India and Pakistan	1910-1950	9.7
Average		10.3

TABLE 1.3

(Cont.)

Commodity and Country:	Period Covered	Year-to-Year Fluctuations[a]
Tin:		
Belgium-Congo	1917-1950	25.9
Bolivia	1902-1950	11.1
Indonesia	1902-1950	19.6
Malaya	1902-1950	18.0
Nigeria	1904-1950	21.2
Thailand	1902-1950	15.7
Average		17.5
Tobacco:		
Algeria	1902-1950	22.6
Indonesia	1902-1938	15.7
Philippines	1904-1950	12.4
Average		15.8
Sugar:		
Cuba	1903-1950	17.0
Indonesia	1902-1950	17.2
Mauritius	1902-1950	17.9
Philippines	1902-1950	23.4
Average		18.3
Average of 10 Commodities	17.98	

Source: Instability in Export Markets of Underdeveloped Countries, op. cit., p. 30 and 31.

[a]Measured as a percentage of the higher value.

The United Nations study on export markets of primary commodities also gives an analysis of the cyclical movements of volume of exports of some commodities.[10] The average duration of each phase of the cycle is found to be under two years, and the amplitude of the cycles around 27.5%, so that on the average these cycles are of shorter duration and smaller amplitude than the price cycles.

A detailed study of the relation between price and volume cycles, we believe, should form the basis of an interesting study. No detailed analysis, as far as we know, has been conducted to determine the relation between these cycles.

A more direct attempt to trace the correlation between year-to-year price volume gives some interesting, though inconclusive, evidence. We give the correlations for 18 commodities in Table 1.4. Of the 18 products, 3 showed a significant positive correlation, while 2 showed a negative correlation, but the majority of the commodities did not suggest a significant relation in either direction. Though these results are inconclusive, there is, however, an interesting relation between industrial raw material and foodstuffs, which merits further study. All the correlations for industrial raw materials tend to be positive and greater than the correlations for foodstuffs, which are negatively correlated. This tends to suggest the following hypothesis: In the case of industrial raw material, prices and volume move in the same direction, which would be true if they are governed by changes in demand, while a negative correlation between prices and volume would mean that

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10/ <u>Instability in Export Markets of Under-developed Countries</u>, <u>op. cit.</u>, pp. 32-35.

TABLE 1.4

CORRELATION BETWEEN YEAR-TO-YEAR FLUCTUATIONS IN

PRICE AND VOLUME

Hemp	+0.598	Wheat	-0.041
Copper	+0.481	Silk	-0.063
Tin	+0.254	Sodium Nitrate	-0.096
Petroleum	+0.110	Tobacco	-0.0976
Wool	+0.089	Coffee	-0.146
Jute	+0.0065	Tea	-0.151
Cotton	-.00143	Cocoa	-0.196
Rubber	-.00288	Rice	-0.308
Sugar	-0.0349	Linseed	-0.4204

Source: Instability in Expert Markets of Underdeveloped Countries, *op. cit.*, p. 58.

prices and volume move in opposite directions, which could arise if the fluctuations occurred as a result of changes in supply as caused by good and bad harvests. Or this suggests the reasonable hypothesis that fluctuations in industrial raw material is caused by changes in demand, while those in foodstuffs by changes in supply.

However, we have not yet any reason to believe that there is any relation between movements in prices and volume. That their combined effect on export earnings has not been to counterbalance one another is seen from the analysis of the fluctuations in export proceeds from primary production. In Table 1.5, we give the year-to-year fluctuations of the export earning of 15 underdeveloped countries, broken down into major export commodities. The extent of fluctuation is measured as before. The average fluctuation of the proceeds is 23.88%, higher than the average year-to-year fluctuations of both prices and volume. Thus there is no tendency of the price and volume fluctuations to counterbalance one another, nor do they synchronize perfectly; for if this were so, the extent of the fluctuations in export proceeds would be much greater. Also, in the study of cycles of export proceeds, the United Nations study finds the fluctuations greater than those of price and volume cycles.[11/]

These fluctuations in export proceeds are of special significance for the underdeveloped countries producing them, because of the large size of the primary producing sector of the economy and because of the great dependence of these countries on foreign trade. The percentage of foreign trade (exports + imports) to total flow of resources

11/ Ibid., p. 14.

TABLE 1.5

YEAR-TO-YEAR FLUCTUATIONS IN EXPORT PROCEEDS BY COUNTRIES

(Average Percentage Fluctuations Per Year)

Commodity and Country		Period Covered	Year-to-Year Fluctuations[a]
Algeria:	Tobacco	1902-1950	22.9
Belgium-Congo	Tin	1917-1950	30.9
Bolivia:	Tin	1902-1950	18.4
Brazil:	Cotton	1904-1950	40.5
	Coffee	1902-1950	19.1
	Cocoa	1910-1950	16.4
	Average		24.6
Ceylon:	Tea	1910-1950	12.3
Chile:	Sodium Nitrate	1914-1950	22.2
	Copper	1902-1950	20.8
	Average		21.3
Cuba:	Sugar	1903-1950	21.3
Egypt:	Cotton	1904-1950	19.5
Gold Coast:	Cocoa	1910-1950	20.3
Indonesia:	Rubber	1911-1950	35.3
	Tin	1902-1950	25.9
	Sugar	1902-1950	24.2
	Tobacco	1902-1938	16.4
	Average		29.0
Malaya:	Rubber	1906-1949	36.5
	Tin	1902-1950	26.5
	Average		33.6
Mauritius:	Sugar	1902-1950	24.1
Nigeria:	Tin	1904-1950	28.1
Peru:	Cotton	1905-1950	26.5
	Copper	1902-1950	19.0
	Average		26.4
Trinidad-and Tobago	Cocoa	1910-1950	25.5
	Average for all Countries		23.88

Source: Instability in Expert Proceeds of Under-Developed Countries, op. cit., p. 43.

[a]Year-to-year changes measured as a percentage of the higher value.

(gross domestic product + imports) for South Rhodesia is as high as 68%, for Belgium Congo, 57%; for Ceylon 55%.[12] This great dependence on foreign trade and economic backwardness makes the primary goods-producing underdeveloped countries especially vulnerable to violent fluctuations.

The effect of these fluctuations is more serious to those primary producing countries specializing in a few products. They cannot rely on the averaging of price movements of many products. Such conditions are found to be true in countries such as Cuba, which depends on sugar, Ceylon on tea, Malaya on rubber and tin, Bolivia on tin. The direct losses arising from fluctuations both in the primary goods-producing sector and the rest of the economy will be discussed in Chapter 3.

Inflation is a very common characteristic of these underdeveloped countries. The United Nations study observes that inflationary pressure is present both during an upswing and during the downswing. "It is a notorious fact that boom conditions in primary product markets tend to induce powerful inflationary pressures in the economies of these countries.... Paradoxically, underdeveloped countries often also experience inflation when foreign demand for their exports is in severe decline."[13]

The non-economic consequences of these fluctuations on the underdeveloped countries are not insignificant. We note the following remarks by Wendall C. Gordon: "Violent fluctuation in sugar prices was a major cause of the revolution in Cuba in the early thirties. Violent

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12/ Commodity Trade and Economic Development, op. cit., p. 8.

13/ Ibid.

fluctuations in the coffee prices were a major cause of the revolution in Brazil in 1930."[14]

As to the future of the fluctuations, we agree with the United Nations report when it says, "There is no reason to think that the instability of commodity prices will disappear of itself. In other words, we reject the view that fluctuations since 1929 have been due to special causes - the great depression, the war and its aftermath, Korean hostilities - which are never likely to occur, and in the absence of which stability would be easily and naturally achieved."[15]

That these violent fluctuations are still with us and need solution is also recognized by Professor Hirschman, when he says, "Fluctuations in commodity prices are still with us; in fact, they appear to be at this writing the one international economic problem which the Western world has not even begun to solve."[16] The existence of the problem and the need for a solution were also noted by Dr. Singer at the conclusion of the second Kyklos Symposium, when he said, "Everybody concerned with the second symposium - and for that matter, the first symposium as well - agrees that in the stabilization of the export markets of underdeveloped countries we have a big unsolved problem. They further agree that we have perhaps made less progress towards the solution of this problem than with any other major problem of our day.

14/ Wendall C. Gordon, The Economy of Latin America, (New York: Columbia University Press, 1950).

15/ Commodity Trade and Economic Development, op. cit., p. 12.

16/ Hirschman, A. O., "Primary Products and Substitutes: Should Technological Progress be Policied," Symposium 2, Kyklos, Vol. 12, 1959, Fasc. 3, p. 354.

They agree that there is need for fresh thinking, for a fresh approach to the problem."[17]

1.3 The Problem.

Economic policy towards the stabilization of fluctuations in primary production could take many different forms. International committees which have studied this problem have suggested many such direct economic policies, both at the national and international levels, towards the stabilization of primary producing underdeveloped countries. Our problem is to look for a solution through the diversification of investments. To put it in another way, the purpose of our study is to determine a rational basis for allocation of investment in a primary producing underdeveloped country, taking the fluctuations in the export markets into special recognition.

The problem indeed is a very real one. It must confront the economic planners of all the primary producing countries. It must, for instance, have confronted the planning secretariat of Ceylon in the formulation of the ten-year plan (1959-1968). This plan hopes to allocate roughly equal amounts of capital in the next ten-year period to the expansion of primary production as to the development of industries.[18] However, it is not clear from the report of the plan how this decision was reached. There has been no attempt made so far, to determine a rational basis for finding how much investment is to allocated to

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[17] Singer, H. W., "Epilogue," Symposium 2, Kyklos, Vol. 12, 1959, Fasc., 3, p. 400.

[18] National Planning Council, The Ten-Year Plan, (Ceylon: Government Press, 1959).

manufacturing industries, how much to be allocated to the expansion of existing primary product industries and how much to the starting of new primary industries. The main purpose of our dissertation is the analysis of this problem.

The discussion of this main problem raises further subsidiary problems. There is the need for an economic model for a primary producing country. Before any investment policy for the whole economy could be discussed, it is necessary to see how these fluctuations that arise in the primary producing sector affect the rest of the economy. Also, it is necessary to determine the nature of the economic losses that these fluctuations bring about, not only in the primary goods sectors, but also the indirect losses that are induced in the rest of the economy.

The particular allocation problem also raises two interesting subsidiary problems: First, there is the question of an allocation theory for underdeveloped countries, which has not been solved to everyone's satisfaction, and there is the question of allocation under uncertainty.

We do not intend treating very extensively all these rather difficult problems; however, we shall deal briefly with each of these problems as they arise in relation to our main problem.

1.4 Plan of Dissertation and Summary.

Before the formal model of the economy is discussed in Chapter 3, we consider it important that the historical setting and the institutional structure of underdeveloped countries be considered. This we do in Chapter 2 of this study. It is our purpose in this chapter to show

the dual character of the type of economies that we intend studying, and some of the special economic institutions that characterize such economies. The case of Ceylon is considered as a typical primary producing country. A brief sketch of her history is considered, to see the origin and character of the dualist nature of these countries; further, also to see some of the basic assumptions of the model that are to be made in the next chapter.

In Chapter 3, the formal model is discussed. The economy is divided into three sectors, which we shall call export, domestic manufacturing, and subsidiary. The export sector is the primary commodity-producing sector; the product of this sector is for export and is characterized by fluctuations in price, output, and proceeds of the sector. The domestic manufacturing sector is that which produces manufacturing goods for domestic consumption. The subsidiary sector, constituting the rest of the economy, is characterized by its pre-capitalist nature. In discussing the working of the model, we determine the effect of the fluctuation arising in the export sector on the other sectors of the economy. We conclude this chapter by discussing the nature of the losses arising in the economy as a result of the fluctuations originating from the export sector.

The rest of the dissertation is devoted to the problem of allocation of resources. We begin Chapter 4 with a discussion on the problem of allocation of resources in underdeveloped countries. We then develop a logical basis, by extending some results of Chenery and Tinbergen, for the analysis of the problem at hand. With the logical framework determined, we discuss the rational procedure for the allocation of resources between the domestic manufacturing and the export sectors.

Chapters 5 and 6 are devoted to the problem of allocation of resources within these two sectors of the economy. For the analysis of these problems, techniques of programming under uncertainty are used. We start Chapter 5 with a discussion of the general problem of programming under uncertainty before going on to the specific problem; we discuss and modify some results of Madansky with regard to the general problem. The rest of Chapter 5 is devoted to the problem of diversifying the export sector; the 'optimum' combination of different primary products.

Chapter 6 is devoted to the problem of allocation among different industries in the domestic manufacturing sector. As mentioned before, the characteristic of this sector is the fluctuation in the demand for the products, arising from fluctuations in the export sector. Here we discuss the problem of allocation of investment among different industries when the demand for the prouuct is subject to random fluctuations. In the final part of this chapter we extend the above results to the case of an inter-industry model in taking into consideration the possibility of the output of the industries being used in the production of other commodities.

CHAPTER 2

HISTORICAL SETTING AND THE INSTITUTIONAL STRUCTURE

2.1 Introduction:

In the analysis of economic systems the study of the economic institutions of the particular system is very fundamental. An economic theory cannot be considered a theory of a particular economic system if the institutions of the system are such, that the premises of the theory are not satisfied. As Boeke remarks, "Every economic system has its own economic theory. A social economic theory is always the theory of special social systems. Even if it announces itself as a general theory still it is historically determined."[1]

In the search for an economic theory for the underdeveloped countries of today, we find economic theorists looking in two directions. First there are the economic theorists who look to neo-classical theories, which they believe, by the proper modification can be extended into an economic theory of the underdeveloped countries of today. A recent book on economic development is called, Keynesian Theory of Economic Development. It is the basic assumption of these economists, that the economic structure and the economic institutions of the underdeveloped countries, though not identical, are similar to those of the mature capitalist countries. Secondly, there are the economists who look to the writings of the classical economists in search of such a

[1] Boeke, J. H., Economics and Economic Policy of Dual Societies: As Exemplified by Indonesia, (New York: Institute of Pacific Relations, 1953), p. 4.

theory. These economists believe that the economic structure of the contemporary underdeveloped countries are similar to those economies that confronted the classical economists - Europe at the time of the industrial revolution.

Undoubtedly, in the contemporary underdeveloped countries, there is much that is common with pre-capitalism and early capitalism, but there is also much that is different. This major difference is due to the fact that these countries have existed side by side with the mature capitalist countries for quite a long time. Most of the underdeveloped countries of today have, at some time of their history, been dominated politically or economically by the mature capitalist countries, and there has always been trade between these countries. These contacts have resulted in the transplantation of some of the economic institutions of the mature capitalist countries. The casual visitor to the under-developed countries is often struck by the multitude of techniques of production and economic institutions that co-exist and compete with one another. In transportation, for instance, one sees that the most primitive methods, coexisting with the most technically advanced and the whole spectrum in between; comparable to a museum of technological history.

Boeke[2] refers to such societies as dual societies. He also uses the term plural society when more than two such distinct social systems coexist. The concept of dual and plural societies as used by Boeke has

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2/ Ibid.

often been misinterpreted.[3] He qualifies the term "by reserving it for societies showing a distinct cleavage of two synchronic and full grown social styles which in the normal, historical evolution of homogeneous societies are separated from each other by transitional forms, as, for instance, precapitalism and high capitalism by early capitalism, and which there do not coincide as contemporary dominating features."[4] A society which is in the process of transition from precapitalism to capitalism, or as he says when "late capitalistic society is gradually superseded by a socialistic system that has grown up internally," he considers as homogeneous rather than a dual society.[5]

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[3] For example, Benjamin Higgins in Economic Development, (New York: Norton & Co., Inc., 1959), says, "there can be no question about the phenomenon of dualism; it is one of the distinguishing features of underdeveloped countries. Virtually all of them have two clearly differenciated sectors: one confined mainly to peasant agriculture and handicrafts or very small industry, and the trading activities associated with them; the other consisting of plantations, mines, petroleum fields and refineries, large-scale industries, and the transport and trading activities associated with these operations. Levels of technique, productivity, and income are low in the first sector and high in the second." Having thus recognised the existence of dualism, he goes on to say, "He (Boeke) thought it had to do with the nature of society, if not the people themselves. As we will see below, dualism is more readily explained in economic and technological terms; and this explanation withstands scrutiny better than Boeke's sociological explanation. It is well such is the case; for if Boeke were right, all our efforts to produce a take-off into sustained growth in underdeveloped countries through vigorous development programs supported by technical and capital assistence from the West would be in vain." It is apparent from the above, that Higgins fails to recognise the difference between feudalist and capitalist social structures that Boeke emphasises so much. Perhaps Boeke is right that unless the feudal social structure of this sector does not change, 'capital assistence from the West would be invain.'

[4] Boeke, J. H., op. cit., p. 3.

[5] In a recent paper, Dale W. Jorgenson, (The Development of Dual Economy," Economic Journal, June 1961, pp. 309-334), uses the term dual economy in considering a model for the transition from precapitalism to capitalism. Clearly this is not the sense in which Boeke used this term, as is apparent from the above quotation.

The origin of dual societies is not social evolution. Boeke makes the following remarks as to its origin, "one of the prevailing systems, as a matter of fact always the most advanced, will have been imported from abroad and have gained its existence in the new environment without being able to oust or to assimilate the divergent social system that has grown up there, with the result that neither of them becomes general and characteristic of that society as a whole. Without doubt the most frequent form of social dualism is to be found there where an imported western capitalism has penetrated into a precapitalist agrarian community and where the original social system—be it not undamaged—has been able to hold its own or, expressed in opposite terms, has not been able to adopt the capitalistic principles and put them into full practice."[6] The inability of the two divergent social systems to assimilate is an important characteristic of social dualism.

Economic structure and institutions vary widely among different underdeveloped countries. However, there are groups of underdeveloped countries which have had similar economic histories and are having similar economic institutions. It is one such group of underdeveloped countries, showing prominent dualistic characteristics, that we intend analysing. These are the countries in which the influence of western capitalism has resulted in the production of primary commodities without changing the social fabric of the indigenous population.

Before discussing the formal model of such an economy in the next chapter, we shall discuss the dualistic character and its historical

[6] Boeke, op. cit., p. 4.

origin together with other basic economic institutions of one such country, Ceylon. The case of Ceylon is not unique; the historical pattern of the development of its capitalist sector is common to many primary producing countries. Countries such as Indonesia, Malaya and many of the Central and South American countries have a very similar economic structure.

2.2 Case of Ceylon

2.2.1 Export Sector and its Origin: The entry of British capitalists to a feudal Ceylon goes quite far back. It was not long after the conquest of the whole island that the British capitalists realised the profitability of the plantations. The Kingdom of Kandy was annexed in 1815, and the first coffee plantations were started in 1825. The capitalists had all the encouragement and help from the new government of the country. In the early stages the land was given free for the plantations and even later only a small fee of five shilling was charged per acre. The legislation necessary for obtaining the land was very willingly enacted by the government.[7/] This early legislation also had

[7/] "If there was any doubt of the Crown's title to lands so lavishly disposed to colonists from England, such doubts were soon resolved by a law passed in 1840—the Crown Lands (Encroachments) Ordinance, No. 12 of 1840. Under this law it was declared that; all forests, waste, unoccupied or uncultivated land was presumed to be the property of the Crown until the contrary was proved,.... in the country chenas and other land which can only be cultivated after intervals of several years, shall be deemed to be forests of waste lands, i.e., shall be presumed to be the property of the Crown until the contrary be proved....

Further accretions of land to the Crown resulted from the operation of the Registration of Temple Lands Ordinance, No. 10 of 1856....

This was followed by the Waste Lands Ordinance, No. 1 of 1897, which gave the Crown the facility of declaring vast tracts of the country as Crown land..." Ceylon Government, Sessional Papers XVIII of 1951: Report of the Kandyan Peasantry Commission, (Colombo; Government Press, 1951), pp. 71-72.

its impact on the social structure. With the removal of much of the land and the feudal services, the feudal lords were deprived much of their previous power and role in society. The advantage of this to the peasantry, as we shall see, was very little. To the capitalists this was both a source of land and of labor.

The rate of investment in this early period was very high. "The mountain ranges on all sides of Kandy became rapidly covered with plantations. It was estimated that 3,000,000 were invested between 1837 to 1845. Capitalists from England arrived by every packet...."[8/] After a minor set back due to the depression of 1847 the coffee industry recovered again by 1853. The coffee industry reached its peak in Ceylon about 1868; the industry was to soon completely disappear from the Island. A coffee blight which appeared in 1868 reduced the area under cultivation by 1870 to about a third. Soon the industry completely disappeared from Ceylon. The area under coffee now is less than 200 acres. The short history of the coffee industry shows well the basic characteristics of a capitalist industry. The rapid expansion of the industry once the innovation proves successful, and the sensitivity of the industry to the business cycle are well seen in its history. The remarkable recovery of the plantation industry after this great disaster also shows the ingenuity of a capitalist enterprise. After experimenting with a few other crops, all of them foreign to the country, tea and rubber were finally introduced in place of coffee.

8/ Sir J. E. Tennent, Ceylon an Account of the Island, Vol. 2, (London: Longman, Green, Longman, and Roberts, 1859), pp. 230-231.

The development of the tea industry, once it was proved successful, was again very rapid. In 1873 the area under tea cultivation was only 200 acres, but by 1890 it had gone up to 220,000 acres, and by 1903 it had gone up to 406,000 acres which constituted about 56% of the total exports of the island, and also a major producer in the world. Rubber, which became the second largest export product of Ceylon, was introduced about the same time as tea; again the sensitivity of the industry to profits was very high. When profits went up, the investments in the industry correspondingly went up, showing the capitalist nature of the industry. The rapid rise of the plantation industry in Ceylon can be seen from Table 2.1.

Almost all the capital, for the investments in the plantations came from England; however, there was also a small group of Ceylonese capitalists who followed the British. The supply of capital was never a problem, the British banks and their branches which were established in Ceylon by this time were always ready to lend to rising and prosperous plantation industry.

The labor from the villages and those left landless by the plantations were not sufficient in the early stages to prevent wages from rising. "At the beginning high wages had to be offered to attract labor to the estates. The English however were not confined by the limits of Ceylon in their search for cheap labor. A plentiful supply was available in the neighbouring possessions of India."[9] Thus as in the case of Malaya, British Guiana and Trinidad, indentured labor came to be introduced into Ceylon. Eventually the major proportion of

[9] Sessional Papers XVIII, op. cit., p. 71.

labor in the plantations turned out to be Indian immigrant labor. Native labor as Table 2.2 shows was always available, though short in some periods. The surplus labor,[10] in more recent times has reached very large proportions.

Besides being cheaper, there are other reasons for the preference of Indian labor by the British capitalists. Labor away from home and having nothing except their labor to sell comes much closer to an industrial proletariat than native labor, who often had some ties with the old village. Again the Government of the country was very cooperative in the bringing of indentured labor from India.[11] Thus much of the labor force came very close to an industrial labor force; and therefore we find that in the case of labor market too, the plantation industry has close resemblance to a capitalist industry.

There were other aspects of the industry which showed its capitalist nature. The organisation and the methods of production and cultivation were often excellent. Research into better types of plants

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10/ The estimates of surplus population of Dr. Sarkar are based on the following two basic assumptions, (a) 'as believed by the Agriculture Department of the Government of Ceylon that 8 acres of land is adequate for a peasant family of 4 or 5 persons to produce an income which will ensure it a standard of living which its members are accustomed.' And (b) '10 per cent of the population to be dependent on non-agricultural occupations.' N. K. Sarkar, The Demography of Ceylon, (Colombo: Government Press, 1957), pp. 220-221.

11/ "The Government itself gave every assistence to facilitate the introduction of South Indian labor. For instance, in 1861, the Government took steps to provide four vessels for the transport of Indian labourers from Rameswaram to Mannar; in 1866 an annual subsidy was granted for three years to a company in Bombay for the regular conveyance of labourers from South Indian ports to Colombo. The Government also took steps to make the conditions of labor on estates attractive to the Indian Tamil. Legislature was passed securing the rights of immigrant labourers." Sessional Papers XVIII, op. cit., p. 70.

TABLE 2.1

AREA UNDER PLANTATIONS

YEAR	ACRES IN '000's
1857	81
1871	196
1881	321
1891	334
1901	460
1911	975
1921	1,093
1946	1,500

Source: N. K. Sarkar: Demography of Ceylon (Colombo: Government Press 1957) p. 210.

TABLE 2.2

ESTIMATED SURPLUS POPULATION* IN THE SUBSISTENCE SECTOR

YEAR	NUMBER OF SURPLUS POPULATION '000's
1871	728
1881	79
1891	660
1901	834
1911	1,202
1921	1,452
1946	2,766

*See footnote 9

Source: _ibid._, p. 221.

and more efficient methods of cultivation were constantly carried out at the institutions of research that came to be established quite early. These institutions had a very high standard of scientific research. These investments in research were very rewarding as seen in the high increase in productivity of this sector.

Thus the plantation sector of the economy was capitalist in almost all its forms. Rather than being a part of Ceylon, it seemed more part of England—perhaps, with the exception of labor who did not have the English factory legislation to protect them.

The importance of this sector to the economy is now very great. Its share in the national income has been steadily increasing. Table 2.3 gives some recent figures on the share of the national product originating in exports.

TABLE 2.3

DOMESTIC EXPORTS AS A PERCENTAGE OF GROSS NATIONAL PRODUCT

YEAR	DOMESTIC EXPORTS AS A PERCENTAGE OF GROSS NATIONAL PRODUCT
1951	38.51
1952	30.64
1953	32.53
1954	34.71

Source: Compiled from data in: Ceylon Government, Ministry of Finance: Economic and Social Development of Ceylon (a Survey) 1926-1954, Colombo: Government Press (1955) p. 129.

Though there are Ceylonese capitalists who have followed the British in the opening up of plantations, the ownership until quite recently remained mainly British. Statistics on ownership are hard to obtain. To quote Dr. Sarkar, "even today the predominant economic influence is exercised by the Europeans. In the tea plantations, export businesses, and banking, European ownership and management predominated. No statistics are available of firms and plantations belonging to or managed by the Europeans, neither do we have any information about the size of European interest in Ceylon."[12]

2.2.2 Subsistence Sector: Next, we shall show that the development of the capitalist agriculture in the last century had very little effect on the rest of the economy. The early legislation, resulting from the conflict of interest of the capitalist and the feudal lords brought some social reforms to the peasantry. But, as we shall see, the life of the peasantry was soon to become much worse.

Even though the feudal lords were deprived of much of their power, the social organisation of the villages did not change very much during this period. The Cambridge agronomist B. H. Farmer has the following observations in his study on the Ceylon peasantry, "as in many oriental societies, the village community is a social group. Although now much affected by outside influences, the Dry Zone[13] Sinhalese village functions as a unit, being largely made up of related families who are

[12] N. K. Sarkar, op. cit., p. 173.

[13] The geography of Ceylon is divided roughly into two regions, the dry zone and the wet zone, rainfall being the main distinguishing feature; the dry zone getting less than 75 inches of rainfall. Almost all the plantations are situated in the wet zone.

largely bound together and to the village by ties of kinship, custom, and religion. This strong social cohesion often finds expression in a marked spirit of co-operation...."

Quite apart from that there are very few estates at all in the Dry Zone, the estates in the Wet Zone do not directly enter into the peasant 'agrarian structure,' since they do not own the land the peasant cultivates; they have, however, a potent influence in other ways...."

"Everywhere in the Wet Zone outside the towns and the estates the society is still essentially a peasant one, clearly derived from the same origin as Dry Zone Sinhalese society."[14]

The villages still retain many of their precapitalist forms of economic activity. A large number of the economic activities still takes place outside of a money economy. Even when money enters the villages, its role is quite different from that of the capitalist sector. Money is used for consumption and not in the production process. Commodities are exchanged for money only to be exchanged again for consumption goods. Money does not here take part in the production process as in capitalist production. This C-M-C role of money, rather than the M-C-M, well illustrates the precapitalist nature of this sector. There is no accumulation of capital in such a social structure.

Table 2.4 gives the area under rice cultivation, the basic crop of this sector, and the population of this sector.

14/ B. H. Farmer, Pioneer Colonisation in Ceylon; A Study in Asian Agrarian Problems. (London: Oxford University Press, 1957) pp. 71, 78 and 81.

TABLE 2.4

POPULATION AND AREA UNDER CULTIVATION (RICE) OF
THE SUBSISTENCE SECTOR

YEAR	AREA UNDER RICE CULTIVATION ACRES ('000's)	POPULATION DEPENDENT ON SUBSISTENCE SECTOR ('000's)
1865	600	-
1875	564	2016 *
1885	604	2273 *
1895	650	2425 *
1901	670	2714
1911	678	3091
1921	798	3349
1946	621	4783

*These figures refer to the years 1871,1881 and 1891 respectively.
Source: Sarkar, Demography of Ceylon op. cit., p. 211 and 216.

As we see the area under rice cultivation remained more or less stationary at 600,000 acres, while the population dependent on this sector almost doubled during this period. Moreover the methods of cultivation remained the same during this period. If output per acre changed it was for the worse. In contrast to the capitalist sector, not only was there no research done on better methods of rice cultivation, there was even no attempt to adopt the better methods of cultivation available elsewhere. With land area remaining the same and the rapid increase of population, the land was divided and sub-divided with the passing of every generation, the size of the plots becoming smaller and smaller and the holdings more and more fragmented. "It remains substantially true that most villages of the Wet Zone have no further room for natural expansion. One consequence of this is that

the village lands are being constantly subdivided into smaller holdings-so small that they cannot be worked economically, at any rate by the only methods which the villager understands.... In the dry zone, apart from the Jaffna peninsula (where there is a plentiful supply of water underground) the deficient rainfall usually admits of only one cultivation season in the year. Thus there is chronic underemployment and, since the area of irrigable land is very limited, there are many landless men.... Even in the irrigated areas crops are often ruined by drought, and it is hardly an exaggeration to say that the dry zone villager normally lives on the edge of destitution. In a good year he can hold his own, in a bad year he starves."[15]

The major contact of the peasantry with capitalist agriculture takes place in the labor market. Though much of the permanent labor force, consisting of immigrant Indian labor and Ceylonese labor, live in the plantations; there is also temporary labor from the villages that is employed on the plantations. These contacts have had many effects on the peasantry.

We have seen that there is a very large surplus population in the subsistence sector of the economy. As a potential labor froce this has its effect on the wage rate. What affects the wages is not the marginal product of this sector, which is zero or very close to zero, but, as Arthur Lewis has pointed out,[16] is the average productivity of

[15] Ceylon Government, Sessional Papers VII: Unemployment in Ceylon, (Colombo, Government Press, 1937), p. 11.

[16] W. Arthur Lewis, 'Economic Development with Unlimited Supply of Labor,' Manchester School, May 1954; reprinted in Economics of Underdevelopment, ed. Agarwala and Singh, (London, Oxford University Press, 1958), pp. 400-449.

this sector. The reason for this lies, in the village and the family structure of these countries. The following observation by Farmer makes this quite clear, "Family solidarity is very strong. Support by the family is an insurance against ill health, old age, or personal disaster. For the same reason, there is in the purana (ancient) village no such thing as unemployment, only underemployment; an individual, back from work in the town, merely joins the family group, each of whom applies less labor to the same land in consequence."[17] The inducement for seeking outside work comes, under such conditions, only when the wage rate is greater than the average productivity of the family returns, and not the marginal returns to the family. But in the case of Ceylon, there was an additional source of labor for the capitalists; in neighboring India, the average subsistence would act only as an upper bound to the wage rate.

That the peasantry would be unwilling to work outside for wages below the average productivity has had its consequences. In this connection Arthur Lewis says, "the fact that the wage level in the capitalist sector depends upon earning in the subsistence sector is sometimes of immense political importance, since its effect is that capitalists have a direct interest in holding down the productivity of the subsistence workers. Thus the owners of plantations have no interest in seeing knowledge of new techniques or seeds conveyed to the peasants, and if they are influential in the government, they will not be found using their influence to expand the facilities for agricultural expansion. They will not support proposals for land settlement, and

[17] Farmer, op. cit., p. 72.

are often instead to be found engaged in turning the peasants off their lands."[18] That the average productivity of the subsistence sector has been reduced by almost half during the period of contact with the Capitalist sector, is good evidence that the observations of Lewis, made in relation to West Indies and Africa, were also true in the case of Ceylon.

The average productivity may have been high in some earlier stage of the development of the capitalist sector to induce the capitalists to bring in indentured labor, but the conditions now are in a very depressing stage, as brought out by the following observations of Farmer, "the pre-war Surveys revealed, for their limited range of villages, mean gross monthly earnings, in cash and kind, and including home production expressed in cash value, varying from only Rs.4.28 (about 6s.5d.) per family in a very poor family in the Puttalam District to Rs.31.57 (about 2.7s.4d.). In most villages the majority of gross earnings were worth less than Rs.15 per family per month, with a mean for all villages in the region of Rs.12 per month, (about 18s.).... The Surveys even found a number of villages which were far from self-supporting in basic foodstuffs, and the surveyors commented on 'the paradox of a peasant economy which is unable even to produce its own food requirements, not to speak of creating a surplus.' It is indeed something of a mystery how such villages managed to survive."[19]

[18] Lewis, op. cit., pp. 409-410.

[19] Farmer, op. cit., pp. 68-69.

Another common feature of the present day villages is their indebtedness. To quote Farmer again, "in the Survey villages, more than 50 per cent of the families, and sometimes over 80 per cent were in debt, to the tune of as much as Rs.200. Most loans were not, as in the case of India, for cultivation purposes, but for the purchase of provisions and household goods. The moneylender was usually a local boutique-keeper or land owner."[20]

With the conditions in the subsistence sector as they are, the capitalists need have no fear of rising wages.

Now that the peasants have come to earn part of their earnings from the plantations, there are other external forces effecting their lives. Depressions that effect the Capitalist countries have now come to effect the lives of these people. The effect of depressions have been thus described in Sessional Paper V of 1939. "Ceylon shared the general economic depression which for several years has afflicted the whole world. There has been a severe fall in the price of its principle commercial products and trade was at a low ebb. The public revenue had decreased and all public works had been drastically cut down. The export of tea and rubber was restricted in an endeavour to raise the price of these commodities and this restriction had caused the closing down of appreciable acreage of estates, particularly among the small holdings.

The effect of the general depression was most marked in the towns. It was also felt in the villages of the wet zone where most of the tea, rubber and coconut plantations are situated. There the people were

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20/ Ibid., p. 69.

deprived of much of the employment to which they had been accustomed. The estates no longer needed the same amount of casual labour for weeding, tapping and plucking. Contractors no longer required their services on public and private works."21/ Thus as Sir Ivor Jennings says, "an economic blizzard which starts in the United States may cause a fall of population and loss of land in the remotest village of Ceylon."22/

As Professor Baran notes, "what resulted was an economic and political amalgamation combining the worst of both worlds - feudalism and capitalism - blocking effectively all possibilities of economic growth."23/

We see therefore, that the subsistence sector though dependent on the capitalist sector for part of its income, did not benefit from it or show any signs of assimilating with it. The villages did not to any appreciable degree share in the new income that was created by the plantations. The social organisation, and the methods of production with in the sector also remained to a great extent the same as it did for centuries before. Thus we have in these countries two distinct forms of social organisation existing side by side, as it were two distinct countries. As Boeke remarks, "Indualist countries, we find precapitalism

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21/ Sir Ivor Jennings, The Economy of Ceylon, (London, Oxford University Press, 1948), p. 68.

22/ Ceylon Government, Sessional Papers V of 1939: Report on the Relief of Distress Due to Sickness and Shortage of Food, (Colombo, Government Press, 1939), p. 8.

23/ Paul A. Baran, "On the Political Economy of Backwardness," Manchester School, (January 1952); reprinted in Agarwala and Singh, op. cit., p. 78.

embodied in the rural 'eastern' social system evan at the present data. The rustic community presents all its characteristics. But here it is brought into direct contact with an imported capitalism in its full bloom, and this it is that gives the amphibious character which is indicated by the term dualism."[24] The distinct difference between the capitalist agriculture and the subsistence sector can also be seen from the patterns of ownership of land. The median size of the tea and rubber plantations is in the region of 500 acres; while 71% of the ownership in the subsistence sector were less than 2 acres. (This latter figure excludes the landless peasants which were as high as 26.3% for the whole island in some districts as high as 41.8%).[25]

In the last decade with changes in the political power, the owners of the plantations no longer being the predominant power group, attempts have been made to resettle peasants in new irrigated land in the dry zone. This however has not changed the structure of this sector to any considerable extent; in any case, these projects of resettlement, where only a mere five acres per family are given, would only result in extending the size of the subsistence sector, rather than changing its social pattern. Therefore it seems that in any analysis of such economies its dualistic characteristics should be fully recognised.

2.2.3 Domestic Manufacturing Sector: The introduction of capitalist agriculture and the various other institutions connected with these industries resulted in the growth of a small urban population,

24/ Boeke, op. cit., p. 14.

25/ Farmer, op. cit., pp. 66 and 85.

constituting ten to fifteen per cent of the population. This urban population and the permanent labor force of the plantations induced the growth of a number of small industries. Such a trend was observed in most of the dualist economies, as Boeke notes, "undeniabley there exist in the social fabric of the dualist countries certain elements which show characteristics attributed to early capitalism."[26] These industries suffered from the same difficulties that wiped out the old handicrafts, competition from more efficiently produced imported commodities. With very little protection given them by the government they had a very miserable existence, showing signs of expansion only during periods artificial restriction of imports as during wars and periods of unusual boom, with conditions reverting back to normal once imports are resumed. In Ceylon there were also other industries connected with the capitalist agricultural sector. These were industries such as the fibre industry, coconut processing industries and rubber goods industries. Technologically these industries were very inferior to their counterparts in the more developed countries. The demands for the products of these industries were mainly domestic. But since the income of the countries was subject to fluctuations, caused by external forces, these industries were also effected by these external fluctuations.

The size of this sector, both from the size of the labor force engaged in it and its share of the national product, was small. For Ceylon in 1957 the percentage of the labor force engaged in industry

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[26] Boeke, op. cit., p. 15.

was 8.6% and its share of the gross domestic product was 7.6%.[27]

It must be mentioned that this sector does not constitute a distinct social system in the representation of Boeke. Historically it may be said to lie somewhere between the two distinct social systems that we have been discussing. Our economy is not plural but dual. For the purpose of our analysis, however, we intend treating this sector separately.

There are three reasons for this distinction. Firstly, the production function of this sector does not include land as an important factor of production as the other two sectors. Secondly, this sector is bound to play an important role in the future development of these economies. The ten year plan for Ceylon, for instance, considers the need for industrialisation quite vital for its future development. It is their contention that the opportunities for absorbing the growing population, even with large irrigation schemes is very limited.[28] They consider import substitution as a good basis for this sector. "The availability of a pre-existing market for industrial products (and of labor) affords a point of commencement in planning for industrial development. In 1957 industrial products of an approximate total value of Rs.1543 million were used in Ceylon for purposes of consumption and

[27] National Planning Council, The Ten-Year Plan, (Colombo, Government Press, 1959), p. 28.

That the percentage of the labor force engaged in industry (8.6%) is greater than its percentage share of the gross domestic product (7.6%), implies that the productivity of the industrial sector for 1957 was below the average productivity for the whole country. It must also be noted that the labor force does not include the subsistence sector.

[28] Ibid., pp. 27-28.

investment. Local production, however, accounted for only Rs.380 million or 15% of this amount. The remaining 85%, amounting to Rs.1163 million in value, was supplied through imports from abroad."[29] The investment in this sector in the ten year plan is 26.7% of the total investments as against 22.9% in agriculture. Since the problem of allocation of resources is a major consideration of our model, it is important to consider this sector separately. The third reason for considering this sector separately is that the demand for the product of this sector is mainly domestic, while the capitalist agriculture sector faces an external market.

We thus break up the economy into three sectors. The subsistence sector and the export sectors showing two distinct and full grown forms of social organisation, the third, the domestic manufacturing sector, a small but an important sector from the point of future development of the country.

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[29] Ibid., p. 29.

TABLE 3.1

MAIN VARIABLES USED IN THE MODEL

	SECTORS		
	Export	Domestic Manufacturing	Subsistence
Output	X_1	X_2	X_3
Land	L	-	-
Capital	K_1	K_2	-
Labor	N_1	N_2	-
Income	Y_1	Y_2	Y_3
Income from Wages	Y_{1w}	Y_{2w}	-
Income from Profits	Y_{1p}	Y_{2p}	-
Income of Nationals	Y_{1N}	-	-
Income of Foreigner	Y_{1F}	-	-
Consumption	C_1	C_2	C_3
Consumption from Wages	C_{1w}	C_{2w}	-
Consumption from Profits	C_{1p}	C_{2p}	-
Savings (from Profits)	S_{1p}	S_{2p}	-
Domestic Demand		d_2	d_3

d_1: Domestic demand for imports.

u: Random variable representing random elements in output.

p: Price of export product - a random variable.

Note: u and p are exogenous to the model.

TABLE 3.2

SUMMARY OF THE MAIN EQUATIONS OF THE MODEL

	SECTOR		
	Export	Domestic Manufacturing	Subsistence
Production Function			
	$X_1 = f(L)\ u$ $L = \kappa_1\ K_1$ $L = \ell_1\ N_1$	$X_2 = \kappa_2\ K_2$ $N_2 = \ell_2\ K_2$	$X_3 = U$
Income	$Y_1 = X_1 p = f(L)\ u \cdot p$ $= f(L)\ z$	$Y_2 = X_2$	$Y_3 = X_3$
Income from Wages	$Y_{1w} = \frac{Lw}{\ell_1}$	$Y_{2w} = \frac{\ell_2}{\kappa_2}$	
Income from Profits	$Y_{1p} = f(L)\ z - \frac{Lw}{\ell_1}$	$Y_{2p} = (1 - \frac{\ell_2 w}{\kappa_2})\ X_2$	
Income from Foreign Investments	$Y_{1F} = (1 - \pi)\ Y_1$		
Income from Domestic Investments	$Y_{1N} = \pi\ Y_1$		
Consumption Function	$C_1 = C_{1w} + C_{1p}$	$C_2 = C_{2w} + C_{2p}$	$C_3 = Y_3$
Consumption from Wages	$C_{1w} = Y_{1w}$	$C_{2w} = Y_{2w}$	
Consumption from Profits	$C_{1p} = \gamma_o + \gamma_1\ Y_{1p}$	$C_{2p} = \gamma_2\ Y_{2p}$	
Savings (from Profits)	$S_{1p} = \pi(Y_{1p} - C_{1p})$	$S_{2p} = Y_{2p} - C_{2p}$	
Domestic Demand		$d_2 = b_{20} + b_{21}\ L + b_{22} f(L)\ z + b_{23}\ X_2$	$d_3 = b_{30} + b_{31}\ L + b_{32} f(L) z + b_{33}\ X_2$

Domestic Demand for Imports $d_1 = b_{10} + b_{11} + b_{12}\ f(L)\ z + b_{13}\ X_2 + v_2 + v_3$

And the identities, $Y_1 + Y_2 + Y_3 - (1-\pi)\ Y_{1p} \equiv C_1 + C_2 + C_3 + S_{1p} + S_{2p}$

CHAPTER 3

THE MODEL

3.1 Introduction.

In the last two chapters we discussed the economic institutions and the structure of the primary producing countries. We saw, that the primary producing industries were transplanted, as it were, in an economy which was basically pre-capitalist in its social structure, and that the development of this primary producing export sector did not change the pre-capitalist nature of the subsistence sector to any appreciable degree. The export sector, however, did find in the subsistence sector a ready source of labor supply. The development of the export sector resulted in the growth of a small capitalist manufacturing sector, producing commodities mainly for domestic consumption. The existence side-by-side of these sectors with different economic and behavioral patterns is a characteristic of most under-developed countries. The relative importance of these sectors vary from country to country.

In some under-developed countries like China, an export sector based on land did not develop to any appreciable degree. In such countries the feudal economy had already utilised extensively all the available land, or if further land was available it was not suitable for the development of a land based export industry. The fluctuations in the markets of primary products do not play a very important role in the economies of such countries. But in countries like Ceylon, Cuba or Malaya where land for primary production was available, an export

industry based on land was developed. The fluctuations arising in primary product markets has come to play an important role not only in the producing sector, but in the economy as a whole. The model that we shall be discussing in this chapter is for such a country.

This model, as mentioned earlier, is to consist of three sectors, each with different economic and behavioral characteristics. We shall discuss the nature of production functions, consumption, savings and income of these different sectors. Then we shall discuss the domestic demand for imports, domestic manufactured goods and the products of the subsistence sector. The response of these functions to fluctuations that arise in the export sector is then discussed. The extent of the induced fluctuations in the domestic manufacturing, and the subsistence sectors of the whole economy is considered. Finally we try to evaluate the direct losses that arise in the economy as a result of these fluctuations.

3.2 Production Functions and Income.

3.2.1 Export Sector. The output of this sector could be classified into two major groups; agricultural products, consisting of items of food like sugar, coffee and tea to raw materials like cotton and rubber, and mining and extractive industries like copper, tin and oil.

The production function of this sector could be said to depend on four basic variables, land, labor and what we shall call 'climatic conditions' (to include rainfall, sun-shine, humidity etc.,). It is important to consider the significance of these variables and the manner in which they enter the production function. We shall argue that it is reasonable to assume that there is little or no substitution taking place among the factors of production.

Of these, by far the most important factor in the production function of the export sector is land. It is the richness of the land and the particular advantage in the production of these commodities that originally attracted investments into this sector. Sugar has its own conditions of soil and climate best suited for its cultivation; so has coffee and tea. These optimum conditions of soil and climate are scarce factors of production. Since the best available land would be first used, returns to land would decrease as the land under cultivation increases. We shall therefore assume, decreasing returns to land in the production function of this sector.

One of our basic assumptions and indeed a fundamental character of under-developed countries is the availability of labor in unrestricted quantities at a very low wage rate. Labor, as such, ceases to be a restricting variable of production. To put it in another way, as long as the other factors are available, labor could be readily had at a given wage rate. Under such conditions substitution between labor and land would have already taken place.[1] Thus as long as labor continues to be non-restricting factor, labor could be assumed to be used in the same ratio with respect to land and capital. This is so, not only for the export sector, but also for the domestic manufacturing sector as well.

It is generally recognised that capital is a scarce factor of production of under-developed countries. What is not recognised is the

[1] In this connection see, Paul A. Samuelson, "Abstract of a Theorem concerning Substitutability in Open Leontief Models," in Activity Analysis of Production and Allocation, (ed.) T. C. Koopmans, (New York, John Wiley, 1951) Chapter VII, pp. 142-146.

imperfections of the money market of these countries. The scarcity of capital is due to a large extent to the bad organisation, and the inability of the subsistence and the domestic manufacturing sectors of the economy to make use of the existing banking and the financial institutions. But this scarcity of capital does not apply to the export sector of the economy. This sector has owned its own capital or has been able to borrow readily from banks and other financial institutions. This is true both of the foreign and domestic investments in the export sectors. Therefore, it is reasonable to assume a constant capital to land ratio for the export sector.

Thus we are left with two effective variables in the production function, land and 'climatic conditions'. The effect of this last variable, we shall assume to be of a random nature, behaving in a known statistical manner. It is also assumed that its effect on the production function is multiplicative rather than additive.

Thus, we express the production function of the export sector in the form,

$$X_1 = f(L)\, u \tag{3.1}$$

where X_1 is the output of the export sector, L the land area under production, and u is a random variable of known probability distribution, representing the random effects of the 'climatic conditions'.

The assumption of diminishing returns gives us the relation,

$$\frac{d^2}{dL^2} f(L) < 0 \tag{3.2}$$

The constant capital-land and the land-labor ratios gives us the two further conditions,

$$L = \kappa_1 K_1 \tag{3.3}$$

and

$$L = \ell_1 N_1 \tag{3.4}$$

where K_1 is the capital investment, N_1 the labor input and κ_1, ℓ_1 are constants. The output of this sector is produced mainly for an export market. Compared to the total external demand, the domestic demand, if any, is assumed to be neglegible.

The process of price formation of primary commodities is rather complex and does not lend easily to generalisations. The emperical evidence on the behavior of prices we discussed in Section 1.2 of Chapter 1. There we concluded that there is no definite emperical evidence as to a relation between prices and output. In our model, therefore, regarding the prices of the export commodity we make the simple assumption, that it is a random variable of known probability distribution.[2] Let the price of this commodity be p. Thus if Y_1 is the income of the

[2] The prices of primary commodities as we discussed in Section 1.2, have a strong tendency for cyclical behavior. The behavior of prices are therefore, not purely random but are also determined by their past history. To be more accurate we should consider the prices to be stochastic variables, this however, would considerably complicate our analysis, moreover we could bring out the points we hope to show from the following simple consideration. The same could be said of the variable u.

export sector we have

$$Y_1 = X_1 \, p_1 = f(L) \, u \cdot p = f(L) \cdot z \tag{3.5}$$

where,

$$z = u \cdot p \tag{3.6}$$

z being a product of two random variables is itself a random variable.

The nature of the wage rate in these countries we already discussed in chapter 2. This we mentioned is constant and is determined by the average productivity of the subsistence sector. If w is the constant wage rate, then from equations (2.4) and (2.5) the total wage bill for the export sector is,

$$Y_{1w} = \frac{Lw}{\ell_1} \tag{3.7}$$

Income from profits Y_{1p} is defined as the difference between total income and wages.

$$\begin{aligned} Y_{1p} &= Y_1 - Y_{1w} \\ &= f(L) \, z - \frac{Lw}{\ell_1} \end{aligned} \tag{3.8}$$

Part of the income of this sector, as we mentioned earlier, is earned by nationals and the rest by non-nationals. Let the proportion of income of this sector earned by nationals be π. So that if Y_{1N} is the income of nationals, and Y_{1F} the income of non-nationals,

we have,

$$Y_{1N} = \pi Y_1 \tag{3.9}$$

$$Y_{1F} = (1 - \pi)\ Y_1 \tag{3.10}$$

Further, it is also assumed that because of the capitalist nature of the industry, that the profits Y_{1p} will be maximised. So that when prices decline the marginal plantations will not be worked, and the workers in these plantations will be discontinued.

$$\frac{d}{dL} Y_{1p} = f'(L)\ z - \frac{w}{\ell} \geq 0 \tag{3.11}$$

where $$\frac{d}{dL} f(L) = f'(L)$$

If $\bar{L}$ denotes the total land area available for production, (or the full capacity in land), this total land area will be under production only if,

$$f'(\bar{L})\ z - \frac{w}{\ell_1} \geq 0 \tag{3.12}$$

or

$$z \geq \frac{w}{\ell_1 f'(\bar{L})} \tag{3.13}$$

Let,

$$\frac{w}{\ell_1 f'(\bar{L})} = \bar{z} \tag{3.14}$$

When $z < \bar{z}$, the marginal land will not be worked, the land area under cultivation in this case is given by the equation,

$$f'(L)\, z = \frac{w}{\ell_1} \qquad z < \bar{z} \tag{3.15}$$

If L^* is the solution of this equation, the land under production can be represented by the relation,

$$\begin{aligned} L &= \bar{L} \quad \text{if} \quad z \geq \bar{z} \\ &= L^* \quad \text{if} \quad z < \bar{z} \end{aligned} \tag{3.16}$$

When $z < \bar{z}$, there will be under utilisation of land capital and the displacement of labor.

3.2.2 Domestic Manufacturing Sector. This is a small sector both from the proportion of population engaged in it and also from its share of the national product. The output of this sector is mainly for domestic consumption. Some of the products of this sector compete with similar imported commodities, others cater for purely domestic tastes, though even these could perhaps be substituted by imports.

The production function of the domestic manufacturing sector is very different from that of the export sector. Land, an important factor in the production function of the export sector does not enter into the production function of this sector at all. The production function of this sector depends only on two factors of production, capital and labor.

The role of labor in this sector is very similar to that of the export sector. To this sector too, labor is available in unrestricted quantities, at a wage rate determined by the subsistence sector. Therefore, we shall assume a constant capital labor ratio for this sector.

We assume the following production function for the domestic manufacturing sector,

$$X_2 = f(K_2) = \kappa_2 \cdot K_2 \tag{3.17}$$

where X_2 is the output of this sector, K_2 the capital stock, and κ_2 is a constant.

The constant capital-labor ratio gives us the relation,

$$N_2 = \ell_2 K_2 \tag{3.18}$$

where N_2 is the labor input and ℓ_2 is a constant.

The price of the commodity of this sector we shall assume to be constant in terms of the product of the subsistence sector, which price we shall take to be unity.

Therefore, the income of this sector is given by the relation,

$$Y_2 = X_2 \tag{3.19}$$

and the wage income

$$Y_{2w} = \ell_2 K_2 w = \frac{\ell_2}{\kappa_2} w X_2 \tag{3.20}$$

and income from profits,

$$(3.21) \qquad Y_{2p} = Y_2 - Y_{2w} = X_2 - \frac{\ell_2}{\kappa_2} w X_2 = (1 - \frac{\ell_2 w}{\kappa_2}) X_2$$

where Y_2, Y_{2w}, Y_{2p} denoted the total income, income from wages and income from profits of this sector respectively.

It is also assumed that the profit rate is positive. i.e.,

$$(3.22) \qquad \tau = (1 - \frac{\ell_2 w}{\kappa_2}) > 0$$

Our assumptions for this sector makes the profit rate constant at all levels of production. This makes the total profits and hence also the incentives for expansion of the industry to depend on aggregate demand for the products of this sector, this seems a fairly reasonable assumption for under-developed countries.

Thus it is assumed that this sector will work at full capacity as long as the aggregate demand for its products is large enough to meet the output. The nature of the demand for these products shall be discussed in Section 3.6 of this chapter.

Let $\bar{X}_2$ denote the output at full capacity, and d_2 the demand. This sector will work at full capacity or not, depending on

$$(3.23) \qquad d_2 \gtrless \bar{X}_2 \qquad \text{respectively.}$$

We shall assume that when,

$$(3.24) \qquad d_2 < \bar{X}_2$$

capacity will be utilised only to the extent of meeting the existing demand for its product.

Thus, if X_2^* denotes the output of this sector when working under full capacity, the output of this sector can be represented in the form,

$$(3.25) \qquad \begin{aligned} X_2 &= \bar{X}_2 && \text{if} \quad d_2 > \bar{X}_2 \\ &= X_2^* && \text{if} \quad d_2 < \bar{X}_2 \end{aligned}$$

When working under full capacity, the utilised capital is given by the relation,

$$(3.26) \qquad K_2^* = \frac{1}{\kappa_2} X_2^* \qquad\qquad d_2 < \bar{X}_2$$

and the labor input by the relation,

$$(3.27) \qquad N_2^* = \ell_2 \, K_2^* = \frac{\ell_2}{\kappa_2} X_2^*$$

3.2.3 Subsistence Sector. The output of this sector is mainly agricultural foodstuffs, however, this sector sometimes also produces handicraft commodities. Much of the economic activity of this sector is not of an organised nature, and is mainly for consumption within the sector.

As discussed elsewhere, this sector has had very little from the technological and institutional developments in the rest of the world. The production techniques of this sector has not changed for centuries,

land is still tilled with wooden ploughs, and the handicraft commodities are produced with the simplest of tools.

The concept of capital as it is understood in the other sectors of the economy, has very little meaning within this sector. Even the addition of capital goods from outside will make little difference, if introduced without any fundamental changes in the social organisation of the sector.

The surplus labor within this sector does not make labor an effective variable in the production function of this sector. It is well known that the marginal product of labor within this sector is very small or zero. The availability of large under-employed population makes the output very insensitive to the addition or removal of labor.

Neither is land an effective variable, in the manner it was in the production function of the export sector. The total land at the disposal of this sector is divided extensively into small plots among the different families. All this land is extensively cultivated. In many of the primary producing countries the land available to the subsistence sector is rather limited, additional land when available is owned by the export sector. For the purpose of our analysis we shall assume that the land available for this sector is limited and does not act as an effective variable.[3]

Therefore, we are left with a single variable 'climatic conditions' for the production function of this sector which is itself

[3] In Table 2.3 of Chapter 2 we give the area under rice cultivation in Ceylon for the last century. This area remained almost constant at about 600,000 acres.

a random variable. Let the production function of this sector be,

$$X_3 = f(u) = U \tag{3.28}$$

where U is a random variable of known probability distribution.

We shall not consider in our model the economic activity connected with internal consumption of this sector. Much of this economic activity takes place outside of a monetary economy and it is difficult to make a formal treatment of such economic activities. Only the income that is derived from economic activity with the rest of the economy is considered. This income arises from two main sources; the income from the sale of output of this sector, and the sale of personal services to the rest of the economy. This latter source of income comes from the temporary demand for labor, for construction of investment projects, public works construction and housing. This demand for labor depends on the economic activity of the rest of the economy. Therefore, we shall assume that the temporary demand for labor is a function of income of the export sector. Let this be proportional to the income of the export sector,

$$Y_{3L} = \epsilon Y_1 \tag{3.29}$$

where ϵ is a small constant.

The output of this sector is given by the equation (3.28). Part of this output is consumed within the subsistence sector, let this be represented by,

$$C_3(Y_3) = \bar{C} + \gamma_3 Y_3 \tag{3.30}$$

where $\bar{C}$ and γ_3 are constants. Therefore, the income from the sale of the output of this sector to the rest of the economy is,

$$U - (\bar{C} + \gamma_3 Y_3) \tag{3.31}$$

Therefore, the income of this sector, received from the rest of the economy is, i.e.,

$$\begin{aligned} Y_3 &= U - \bar{C} - \gamma_3 Y_3 + \epsilon Y_1 \\ Y_3 + \gamma_3 Y_3 &= U - \bar{C} + \epsilon Y_1 \\ Y_3 &= \frac{1}{1 + \gamma_3} [U - \bar{C}] + \frac{\epsilon}{1 + \gamma_3} Y_1 \end{aligned} \tag{3.32}$$

3.3 Consumption Function.

3.3.1 Export Sector. We shall consider the consumption function of this sector in two parts; the consumption from wages and the consumption from profits. i.e.,

$$C_1 = C_{1w} + C_{1p} \tag{3.33}$$

where C_1, C_{1w} and C_{1p}, represent total consumption, consumption from wages and consumption from profits respectively.

It is assumed that the whole of the income from wages is consumed. i.e.,

$$C_{1w} = Y_{1w} \tag{3.34}$$

Of the consumption from profits what is relevant to our analysis is only that part earned by the nationals. It is assumed that the income from foreign investments are remitted outside the country, and hence that the consumption from this part of the income does not take place within the economy.

Income from profits will obviously be a determinant of this consumption function. These incomes as we have mentioned before are subject to rather severe fluctuations. We believe that the extent of these fluctuations too are a determinant of the consumption patterns from profits. The greater the fluctuations the greater would be the highest income reached, and the consumption habits formed at these high incomes would continue, even when income drops from this high level. Thus the chances of being exposed to more expensive consumption patterns would be greater, greater the extent of fluctuations of income.

Therefore, we assume a consumption function of the following form,

$$C_{1p} = \gamma_o(\sigma^2_{Y_1}) + \gamma_1(\sigma^2_{Y_1})\ Y_{1p} \tag{3.35}$$

where γ_o and γ_1 are both increasing functions of the 'extent of fluctuations'.

3.3.2 Domestic Manufacturing Sector. Again we divide the consumption function into two parts, the consumption from wages and the consumption from profits. i.e.,

$$(3.36) \qquad C_2 = C_{2p} + C_{2w}$$

As before, it is assumed that the whole of the wage income is consumed. i.e.,

$$(3.37) \qquad C_{2w} = Y_{2w}$$

The consumption function of profits we assume depend only on its income. Though the profits of this sector are also subject to fluctuations, they are not as violent or significant as those of the export sector. This consumption function is of the form,

$$(3.38) \qquad C_{2p} = \gamma_2 Y_{2p}$$

3.3.3 Subsistence Sector. The consumption of the subsistence sector of its own product we have already given in equation (3.31). We shall assume that the whole of its income received from outside is spent on consumption goods bought from outside. That this sector does not save a any of its income from outside is well known.[4/] Thus we have,

$$(3.39) \qquad C_3 = Y_3$$

4/ See Section 2.2.2 of Chapter 2.

3.3.4 Consumption Function for the Economy. Combining the consumption functions for the different sectors, we get the following consumption function for the whole economy.

$$
\begin{aligned}
C &= C_1 + C_2 + C_3 \\
&= \gamma_o + \pi\,\gamma_1(Y_1 - Y_{1w}) + \gamma_2(Y_2 - Y_{2w}) + Y_{1w} + Y_{2w} \\
&\qquad + \frac{\epsilon}{1 + \gamma_3} Y_1 + \frac{1}{1 + \gamma_3}(U - \bar{C}) \qquad (3.40) \\
&= (\gamma_o + \frac{U - \bar{C}}{1 + \gamma_3}) + (\pi\,\gamma_1 + \frac{\epsilon}{1 + \gamma_3})\,Y_1 + \gamma_2\,Y_2 + (1 - \pi\gamma_1)Y_{1w} + (1-\gamma_2)Y_{2w}
\end{aligned}
$$

3.4 Consumption Patterns.

Next we shall consider the breakdown of the domestic consumption into its main sources; imports, domestic manufactured goods and the products of the subsistence sector. The output of the subsistence sector it is assumed is an inferior good, so that as income rises the demand for these goods will decrease. Further it is assumed that the proportion of imported goods in total consumption will increase with income. It is also assumed that excess demand for domestically originating commodities could be met from imports.

We shall group the consumption functions from wage income; it will be assumed that the distribution of consumption from wage incomes of both the sectors are the same.

Let the total consumption from wages C_w consist of imports, domestically manufactured commodities and products of the subsistence sector in the following proportions.

$$(3.41) \qquad C_{w1} = \alpha_{w1} C_w \; ; \quad C_{w2} = \alpha_{w2} C_w \; ; \quad C_{w3} = \alpha_{w3} C_w$$

where $\alpha_{w1}, \alpha_{w2}, \alpha_{w3}$ are considered to be constants in our model, but would in general be functions of income. Also $\alpha_{w1} + \alpha_{w2} + \alpha_{w3} = 1$.

The income from profits are also grouped together, this is assumed to comprise of imports, domestically manufactured, and subsistence goods in the following proportions.

$$(3.42) \qquad C_{p1} = \alpha_{p1} C_p \; ; \quad C_{p2} = \alpha_{p2} C_p \; ; \quad C_{p3} = \alpha_{p3} C_p$$

where the α's are constants as before. Also $\alpha_{p1} + \alpha_{p2} + \alpha_{p3} = 1$.

Since we have already separated the consumption of the subsistence sector of its own product, we will consider here only the separation of C_s into imports and domestically manufactured commodities. Let these be in the following proportions,

$$(3.43) \qquad C_{s1} = \alpha_{s1} \; : \quad C_{s2} = \alpha_{s2} C_s$$

where the α's are same as before, and $\alpha_{s1} + \alpha_{s2} = 1$.

3.5 Savings Function.

In our model savings could arise only from profits. The total of wage income and the income from the subsistence sector is assumed to be consumed. Also from the income from profits only the savings of residents interest us. Even if the savings arising from foreign investments are invested within the economy, these would be still equivalent to foreign investments.

Having already defined the consumption function, we get the savings function as a residual. We get the following savings function for the export sector,

$$(3.44) \qquad S_{1p} = \pi[Y_{1p} - C_{1p}]$$

and the following from the domestic manufacturing sector,

$$(3.45) \qquad S_{2p} = Y_{2p} - C_{2p}$$

Therefore, combining the above two savings functions, we get the following savings function for the whole economy.

$$(3.46) \qquad \begin{aligned} S &= S_{1p} + S_{2p} \\ &= \pi\, Y_{1p} + Y_{2p} - (\pi\, C_{1p} + C_{2p}) \end{aligned}$$

3.6 Demand Functions.

Next, we shall determine the demand functions for imports, domestically manufactured commodities, and the products of the subsistence sector.

From equations (3.33), (3.35), (3.37), (3.38), (3.39), (3.41), (3.42) and (3.43), we get the following demand functions. For imports

$$
\begin{aligned}
d_1 &= \alpha_{w1} C_w + \alpha_{p1} C_p + \alpha_{s2} C_s + v_2 + v_3 \\
&= \alpha_{w1} \left[\frac{Lw}{\ell_1} + \frac{\ell_2}{\kappa_2} w X_2\right] + \alpha_{p1}\{\pi[\gamma_o + \gamma_1(f(L)\ z - \frac{Lw}{\ell_1})] + \gamma_2(1 - \frac{\ell_{2w}}{2})\} X_2 \\
&\qquad + \alpha_{s1}\{\frac{U - \bar{C}}{1 + \gamma_3} + \frac{\epsilon}{1 + \gamma_3} f(L)\ z\} + v_2 + v_3 \qquad (3.47) \\
&= \alpha_{p1} \pi\gamma_o + \alpha_{s1} \frac{(U-\bar{C})}{1 + \gamma_3} + L(\frac{\alpha_{w1} w}{\ell_1} - \frac{\alpha_{p1}\pi\gamma_1 w}{\ell_1}) + f(L)\ z(\alpha_{p1}\pi\gamma_o + \frac{\alpha_{s1}\epsilon}{1 + \gamma_3}) \\
&\qquad + X_2(\alpha_{w1} \frac{\ell_2}{\kappa_2} w + \gamma_2(1 - \frac{\ell_{2w}}{\kappa_2})) + v_2 + v_3 \\
&= b_{10} + b_{11} L + b_{12} f(L) \cdot z + b_{13} X_2 + v_2 + v_3
\end{aligned}
$$

where v_2 and v_3 represent the excess demand, if any, for the domestic manufacturing sector and subsistence sector respectively.

For domestically manufactured goods,

$$
\begin{aligned}
d_2 &= \alpha_{w2} C_w + \alpha_{p2} C_p + \alpha_{s2} C_s \\
&= \alpha_{w2}\left[\frac{Lw}{\ell_1} + \frac{\ell_2}{\kappa_2} w X_2\right] + \alpha_{p2}\{\pi[\gamma_o + \gamma_1(F(L)\ z - \frac{Lw}{\ell_1})] + \gamma_2(1 - \frac{\ell_{2w}}{\kappa_2}) X_2 \\
&\qquad + \alpha_{s2}\{\frac{1}{1 + \gamma_3} [U - \bar{C}] + \frac{\epsilon}{1 + \gamma_3} f(L)z\} \qquad (3.48) \\
&= [\alpha_{p2}\pi\gamma_o + \alpha_{s2} \cdot \frac{U - C}{1 + \gamma_3}] + L(\alpha_{w2} \frac{w}{\ell} - \frac{\alpha_{p2}\pi\gamma_1 w}{\ell_1}) + f(L) \cdot z \cdot (\alpha_{p2}\pi\gamma_1 + \frac{\alpha_{s2}\epsilon}{1 + \gamma_3}) \\
&\qquad + X_2(\frac{\alpha_{w2}\ell_2 w}{\kappa_2} + \gamma_2(1 - \frac{\ell_{2w}}{\kappa_2})) \\
&= b_{20} + b_{21} L + b_{22} f(L)\ z + b_{23} x_2 .
\end{aligned}
$$

For the products of the subsistence sector,

$$
\begin{aligned}
d_3 &= \alpha_{w3}\, C_w + \alpha_{p3}\, C_p \\
&= \alpha_{w3}\left[\frac{Lw}{\ell_1} + \frac{\ell_2}{\kappa_2}\, w\, X_2\right] + \alpha_{p3}\{\pi[\gamma_o + \gamma_1(F(L)z - \frac{Lw}{\ell_1})]\} \\
&\qquad + \gamma_2(1 - \frac{\ell_2 w}{\kappa_2}\, X_2\} \qquad (3.49) \\
&= \alpha_{p3}\, \pi\gamma_o + L\left[\frac{\alpha_{w3} w}{\ell_1} - \frac{\alpha_{p3}\pi\gamma_1 w}{\ell_1}\right] + f(L)\, z\, (\alpha_{p3}\, \pi\gamma_1) \\
&\qquad + X_2\left[\alpha_{w3}\, \frac{\ell_2}{\kappa_2}\, w + \gamma_2(1 - \frac{\ell_2 w}{\kappa_2})\right] \\
&= b_{30} + b_{31}L + b_{32}\, f(L)z + b_{33}\, X_2 \; .
\end{aligned}
$$

where the b_{ij} are constants.

3.7. Fluctuations in the Economy.

3.7.1 Fluctuations in the Export Sector. In Section 3.7 we shall consider the fluctuations arising in the different sectors of the economy. These fluctuations arise as we have mentioned before as a result of the fluctuations originating in the export sector.

We shall first consider the extent of the fluctuations in the export sector. As we have seen in Section 1.2 of the first chapter the fluctuations in the export sector arise from two sources; those from prices, p, and from output, u. We have assumed that p and u are independently distributed random variables of known probability distribution.[5]

[5] Through most of our analysis in this and the following chapters we consider the variance as a measure of the extent of fluctuations. However, due to the stochastic nature of the variable (see footnote 2) the variance may not be the 'best' measure. The variance however has the advantage of being easy to handle.

Let the mean and variance of p be,

$$E(p) = \bar{p} \qquad \text{var } p = \sigma_u^2$$

and the mean and variance of u be,

$$E(u) = \bar{u} \qquad \text{var } u = \sigma_u^2$$

We defined z in equation 3.6 as,

$$(3.6) \qquad z = p \cdot u$$

The probability distribution of z is given by,

$$(3.50) \qquad q(z') = \text{prob}(z = z') = \int_0^\infty \text{prob}(u = u') \cdot \text{prob}[p = \frac{z'}{u'}]du'$$

The mean value of z is,

$$(3.51) \qquad \mu = E(z) = \bar{u}\,\bar{p}$$

and the variance of z is,

$$(3.52) \qquad \text{Var } z = \sigma^2 = \sigma_p^2 \cdot \sigma_u^2 + \bar{u}^2\,\sigma_p^2 + \bar{p}^2\,\sigma_u^2 .$$

The variance of the total proceeds, we see, is greater than the variance of price of the output. Thus they reinforce each other, a result that has been empirically observed.[6]

[6] See Table 1.3 of Chapter 1.

From equations (3.5) and (3.16) the income of the export sector can be expressed in the form,

$$(3.53)\qquad \begin{aligned} Y_1 &= z\ f(L^*) \qquad \text{for} \qquad z < \bar{z} \\ &= z\ f(\bar{L}) \qquad \text{for} \qquad z > \bar{z} \end{aligned}$$

Therefore, we get the value of the export sector in the form,

$$(3.54)\qquad \begin{aligned} E(Y_1) &= \int_0^{\bar{z}} z\ f(L^*)\ q(z)\ dz + \int_{\bar{z}}^{\infty} z\ f(\bar{L})\ q(z)\ dz \\ &= \int_0^{\infty} z\ f(\bar{L})\ q(z)\ dz - \int_0^{\bar{z}} [f(\bar{L}) - f(L^*)]z\ q(z)\ dz \\ &= \mu\ f(\bar{L}) - \delta \end{aligned}$$

where

$$(3.55)\qquad \delta = \int_0^{\bar{z}} [f(\bar{L}) - f(L^*)]\ z\ q(z)\ dz$$

and the variance of the income of this sector is,

$$(3.56)\qquad \begin{aligned} \text{Var } Y_1 &= \int_0^{\infty} [z\ F(L) - (\mu\ f(\bar{L}) - \delta)]^2\ q(z)\ dz \\ &= [f(\bar{L})]^2\ \sigma^2 - \delta^2 + 2\mu\ f(\bar{L})\ \delta - \int_0^{\bar{z}} \{[f(\bar{L})]^2 - [f(L^*)]^2\} z^2 q(z) dz \\ &= [f(\bar{L})]^2\ \sigma^2 - \delta^2 + 2\mu\ f(\bar{L})\ \delta - \delta' \end{aligned}$$

where,

$$(3.57) \qquad \delta' = \int_0^{\bar{z}} \{[f(\bar{L})]^2 - [f(L^*)]^2\} \, z^2 \, q(z) \, dz \; .$$

From equation (3.7) and (3.16) the wage income of this sector can be written in the form,

$$(3.58) \qquad \begin{aligned} Y_{1w} &= \frac{L^*w}{\ell_1} && \text{for} \quad z < \bar{z} \\ &= \frac{\bar{L}w}{\ell_1} && \text{for} \quad z > \bar{z} \end{aligned}$$

Hence the expected value of wage income is,

$$(3.59) \qquad \begin{aligned} E[Y_{1w}] &= \int_0^{\bar{z}} \frac{L^*w}{\ell_1} \, q(z) \, dz + \frac{\bar{L}w}{\ell_1} \int_{\bar{z}}^{\infty} q(z) \, dz \\ &= \frac{\bar{L}w}{\ell_1} - \int_0^{\bar{z}} (\bar{L} - L^*) \frac{w}{\ell} \, q(z) \, dz \\ &= \frac{\bar{L}w}{\ell_1} - \frac{w}{\ell} \delta_w \end{aligned}$$

where,

$$(3.60) \qquad \delta_w = \int_0^{\bar{z}} (\bar{L} - L^*) \, q(z) \, dz \; .$$

From equation (3.8) and (3.16) the income from profits can be written as,

$$(3.61) \qquad \begin{aligned} Y_{1p} &= z \, f(L^*) - \frac{L^*w}{\ell_1} && \text{for} \quad z < \bar{z} \\ &= z \, f(\bar{L}) - \frac{\bar{L}w}{\ell_1} && \text{for} \quad z > \bar{z} \end{aligned}$$

Hence the expected value of income from profits is,

$$E[Y_{1p}] = \int_0^{\bar{z}} [z\ f(L^*) - \frac{L^*w}{\ell_1}]\ p(z)\ dz + \int_{\bar{z}}^{\infty} [z\ f(\bar{L}) - \frac{\bar{L}w}{\ell_1}]\ q(z)\ dz$$

(3.62)

$$= \bar{p}\ \bar{u}\ f(\bar{L}) - E[Y_{1w}] - \delta$$

We see that expected value of profits depend not only on the expectations $\bar{p}$ of price and $\bar{u}$ of output, but also on δ. Note that δ will be positive if the probability that the export sector works under full capacity

(3.63) $$P_{\bar{z}} = \mathrm{Prob}[\ z < \bar{z}]$$

is positive.

3.7.2 Fluctuations in the Domestic Manufacturing Sector. Next, we shall consider the fluctuations induced in the domestic manufacturing sector, by fluctuations arising from the export sector.

The demand for the product of this sector is given by equation (3.48), this is expressed as a function of the income of the export sector $f(L)z$, and L the land under cultivation of the export sector. From equations (3.53) and (3.16), we see that these variables are dependent on the random variable z. Therefore, the demand equation (3.48) is itself dependent on this random variable, which makes d_2 a random variable.

The relation between the demand d_2 and the output X_2 is given by the equation (3.25).

$$
\begin{aligned}
X_2 &= \bar{X}_2 \quad \text{for} \quad d_2 > \bar{X}_2 \\
&= X_2^* \quad \text{for} \quad d_2 < \bar{X}_2
\end{aligned}
$$

We next consider the relation between output X_2 and the random variable z. Let $\bar{\bar{z}}$ be the value of z for which

$$d_2 = \bar{X}_2$$

For high enough values of z both the export and the domestic manufacturing sectors will be working under full capacity. Which of these would first start to work under full capacity would depend on whether condition (3.16) or (3.23) is first reached.

We will first suppose that condition (3.23) is reached before (3.16); i.e., that the domestic manufacturing industry will start to produce under full capacity before the export sector does so.

So that we will have,

$$\bar{\bar{z}} > \bar{z}$$

The output of the domestic manufacturing sector under this condition can be represented in the form,

$$
\begin{aligned}
X_2 &= \bar{X}_2 \qquad\qquad \text{for} \quad z > \bar{\bar{z}} \\
(3.64) \qquad &= X_2^* = \frac{1}{(1 - b_{23})} [b_{20} + b_{22} f(\bar{L}) z + b_{21} \bar{L}], \quad \text{for} \quad \bar{z} < z < \bar{\bar{z}} \\
&= X_2^{**} = \frac{1}{(1 - b_{23})} [b_{20} + b_{22} f(L^*) z + b_{21} L^*], \quad \text{for} \quad z < \bar{z}
\end{aligned}
$$

Thus for values of $z > \bar{\bar{z}}$ the output would be constant, and for $\bar{z} < z < \bar{\bar{z}}$ the output depends linearly on z, and for $z < \bar{z}$ the decline of output would be non-linear and much more sharp.

In the other case, when the export sector starts to produce under full capacity before the domestic manufacturing sectors, i.e., when

$$\bar{\bar{z}} < \bar{z} \ .$$

The output of the domestic manufacturing sector could still be represented by equation (3.64), except that we would now have $X^* = 0$.

The expected value of the income of this sector is given by,

$$E(X_2) = \int_0^{\bar{z}} X_2^{**} \, q(z) \, dz + \int_{\bar{z}}^{\bar{\bar{z}}} X_2^* \, q(z) \, dz + \int_{\bar{\bar{z}}}^{\infty} \bar{X}_2 \, q(z) \, dz$$

$$= \bar{X}_2 - \int_{\bar{z}}^{\bar{\bar{z}}} (\bar{X}_2 - X_2^*) \, q(z) \, dz - \int_0^{\bar{z}} (\bar{X}_2 - X_2^{**}) \, q(z) \, dz \tag{3.65}$$

$$= \bar{X}_2 - \delta_2 - \delta_3$$

where,

$$\delta_2 = \int_{\bar{z}}^{\bar{\bar{z}}} (\bar{X}_2 - X_2^*) \, q(z) \, dz \tag{3.66}$$

and

$$\delta_3 = \int_0^{\bar{z}} (\bar{X}_2 - X_2^{**}) \, q(z) \, dz \ . \tag{3.67}$$

δ_2 and δ_3 are both non negative. From equation (3.19), (3.20) and (3.21) we see that the wage income and the income from profits are proportional to the total income of this sector. Therefore the fluctuations

in wage income and income from profits are proportional to the fluctuations in total income. So that the expectations and the variance of wages and profits would themselves be proportional to those of total income.

It must be noted that δ_2 and δ_3 are dependent on the range of fluctuation of z. Larger the range of fluctuation, larger would be δ_2 and δ_3, and hence, lower the income of this sector. It could therefore be said that greater the extent of fluctuation of z the less would be the income of this sector. Thus the incentives for expansion of the domestic manufacturing sector would also depend on the extent of fluctuation of z.

Therefore, we could conclude that the extent of fluctuations in the export sector are very vital to the economic activity of the domestic manufacturing sector.

3.7.3 Fluctuations in the Subsistence Sector. The income of the subsistence sector is given by equation (3.32) as,

$$Y_3 = \frac{1}{1 + \gamma_3} [U - \bar{C}] + \frac{\epsilon}{1 + \gamma_3} Y_1 \tag{3.32}$$

This income, we see, depends on two random variables, the output of this sector and the income of the subsistence sector.

The expected value of this income is,

$$E[Y_3] = \frac{1}{1 + \gamma_3} [U - \bar{C}] + \frac{\epsilon}{1 + \gamma_3} E(Y_1) \tag{3.68}$$

and the variance is,

$$\text{(3.69)} \qquad \text{Var } Y_3 = \frac{1}{(1 + \gamma_3)^2} \text{ Var } U + \left(\frac{\epsilon}{1 + \gamma_3}\right)^2 \text{ Var } Y_1$$

The effect of the fluctuation arising in the export sector on the subsistence sector, we see from the above equation, depends on ϵ. i.e., it depends on how large a proportion of the income of this sector is received by the sale of labor to the export sector. If we assume that the variations in U are comparable to those in u, then, since the variation in Y_1 is greater than that in u,[7] the fluctuations in Y_1 would be larger than those in U. So that even when ϵ is small the effect of the fluctuations arising in the export sector on the subsistence sector are comparable to those that originate within this sector.

Since we have assumed that the whole of the income of this sector is consumed, the fluctuations in the consumption of this sector are the same as those of income.

The demand for the products of the subsistence sector is given by equation (3.49). For the consistency of the model, we shall assume that the difference between this demand and the supply, which is the same as Y_3, is either freely disposed or met with imports.

3.7.4 <u>Fluctuations in Total Income</u>. Having briefly discussed the behavior of the different sectors of the economy, we next discuss the fluctuations in the total income Y.

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[7] See Table 1.3 of Chapter 1.

$$Y = Y_1 + Y_2 + Y_3$$

(3.70)
$$= f(L)\ z + X_2 + \frac{1}{1+\gamma_3}[U - \bar{C}] + \frac{\epsilon}{1+\gamma_3} f(L)\ z$$

$$= \frac{1}{1+\gamma_3}[U - \bar{C}] + (1 + \frac{\epsilon}{1+\gamma_3}) f(L)\ z + X_2$$

The behavior of the total income as a result of the fluctuations in the export sector can be considered from the above equation. This is best seen by considering the behavior of (3.70) during the down-ward phase of z.

For values of $z > \bar{\bar{z}}$, all the elements other than z of this function would be constant. So that in this range of variation of z, the variation of the total income would be linearly dependent to z. But for z such that $\bar{z} < z < \bar{\bar{z}}$, variations in z would cause variations in X_2 as well, as seen from equation (3.64). Therefore, as z varies in this range, the corresponding variation in Y would be more rapid, because the variations in z would now be combined with the variations in X_2. As z declines beyond $\bar{z}$, not only will X_2 decline much faster than that of the previous period (see 3.64), but $f(L)$ will also now decline with z. Therefore, during this last phase a decline in z would now cause very severe decline in the total income. Thus we see that during depression in the export sector the effect in the economy could be very severe, its effect on the total economy is of a much greater than would be suggested by considering the fluctuations in z alone.

3.7.5 Behavior of Imports. Next, we consider the behavior of imported consumption goods in the above model. The demand for imported goods for consumption is given in equation (3.47).

It has been shown that the demand for imported goods is dependent on the fluctuations of the export sector. There are three reasons why it is likely that the consumption from imports is rather high in primary producing countries. (a) Fluctuations result in a decline of the domestically manufacturing industry. Therefore, products which otherwise would be met domestically would now be met from imports. (b) Consumption habits formed at high levels of income for better finished imported goods do not completely disappear when income declines. (c) At high levels of income, when the demand for domestically produced commodities exceeds supply, the excess demand is usually met with imports. Therefore, beyond a certain level of income, almost all the increase in demand for consumption would be met from imports. It may therefore be said that greater the fluctuations in income, the greater the tendency to depend on imported consumption goods.

The foreign exchange earnings, on the assumption that the profits from foreign investments are remitted out of the country, is given by the following equation,

$$
\begin{aligned}
F = Y_1 - (1 - \pi)\, Y_{1p} &= f(L)\, z - (1 - \pi)\, [f(L)\, z - \frac{Lw}{\ell_1}] \\
&= \pi\, f(L)\, z + (1 - \pi)\, \frac{Lw}{\ell_1}
\end{aligned}
\tag{3.71}
$$

The variations in this function too can be considered in the same way as above, by considering the variations that arise from L, f(L) and z.

It has been observed in emperical studies in some Latin American countries, especially in Mexico,[8] that the income elasticity of imports is greater than unity. i.e., a proportionate increase in income would increase imports by a greater proportion. We shall obtain the income elasticity of imports for large values of income for our model, and show that is is quite possible for the above empirical observation to be true.

For values of z large enough, both the export and the domestically manufacturing sector would be working at full capacity. In this range the total increase in income goes either as profit of the export sector or as income of the subsistence sector. And the increase in consumption is met from imports. Under these conditions, we have from equation (3.40),

$$\frac{\Delta C}{\Delta Y} = \frac{\Delta C_I}{\Delta Y_1} = \gamma_1 \pi + \frac{\epsilon}{1 + \gamma_3} \tag{3.72}$$

Therefore, the income elasticity for imports for large enough values is given by

$$e_I = \frac{Y}{C_I} \cdot \frac{\Delta C_I}{\Delta Y} = \frac{Y}{C_I} [\gamma_1 \pi + \frac{\epsilon}{1 + \gamma_3}] \tag{3.73}$$

It is quite possible for this function to be greater than unity.

3.8 The Nature of Losses to the Economy.

We have seen how the economic activities of primary producing countries are dependent on the fluctuations of the export sector. Next

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[8] United Nations, ECLA, External Disequilibrium in the Economic Development of Latin America: The Case of Mexico, (New York, 1957) pp. 62-63.

we shall discuss the losses that arise as a result of these fluctuations.

(a) First, there are the losses arising from the under-utilisation of capital and land and the displacement of labor, in the export sector. We saw that the maximisation of profits would lead to working some industries in this sector under capacity, when $z < \bar{z}$. Though this maximises individual profits, they cause losses to the economy. Displaced labor has to maintain themselves by borrowing, by the aid of the state or if they still have contact with their families in the subsistence sector, with the help of these families. The displacement of labor is certainly a social loss. In the case of utilisation of labor and capital, there would be cost in maintaining plant and land in working condition, so that there is no loss of efficiency when they are re-utilised.

We shall assume that these losses in land, capital and labor are proportional to the extent they are under utilised, then the expected losses in this case is,

$$
\begin{aligned}
L_1 &= \int_0^{\bar{z}} \nu_1 \cdot (\bar{L} - L^*)\, p(z)\, dz + \int_0^{\bar{z}} \nu_2 \cdot (\bar{K}_1 - K_1^*)\, p(z)\, dz \\
&\quad + \int_0^{\bar{z}} \nu_3 (\bar{N} - N^*)\, p(z)\, dz \\
&= \left(\nu_1 + \frac{\nu_2}{\kappa_1} + \frac{\nu_3}{\ell_1}\right) \int_0^{\bar{z}} (L - L^*)\, p(z)\, dz \\
&= \left(\nu_1 + \frac{\nu_2}{\kappa_1} + \frac{\nu_3}{\ell_1}\right) \delta_w
\end{aligned} \tag{3.74}
$$

where ν_1, ν_2 and ν_3 are constants, and δ_w as before.

(b) There are similar losses arising from the under-utilisation of resources in the domestic manufacturing sector. Again, if we assume

that the losses arising from such under-utilisation of capital and labor are proportional to the extent that they are under utilised, we get the following expected loss function,

$$L_2 = \int_0^{\bar{z}} \nu_4(\bar{K}_2 - K_2^{**}) \cdot p(z)\, dz + \int_{\bar{z}}^{\bar{\bar{z}}} \nu_4(\bar{K}_2 - K_2^{*})\, p(z)\, dz$$

$$(3.75) \qquad + \int_0^{\bar{z}} \nu_5(\bar{N}_2 - N_2^{*})\, p(z)\, dz + \int_{\bar{z}}^{\bar{\bar{z}}} \nu_5(\bar{N}_2 - N_2^{*})\, p(z)\, dz$$

$$= \frac{\nu_5}{\kappa_2}(\delta_2 + \delta_3) + \frac{\kappa_2 \nu_5}{\ell_2}(\delta_2 + \delta_3)$$

$$= (\frac{\nu_4}{\kappa_2} + \frac{\kappa_2 \nu_5}{\ell_2} (\delta_2 + \delta_3)$$

where ν_4 and ν_5 are constants, and δ_2, δ_3 as before.

(c) We saw that the profits were dependent not only on expectations of z, but also on the extent of its fluctuations, and that there is a reduction in profits depending on the extent of these fluctuations. This decline in profits has consequences (other than the immediate effect of reduction of income), on the economy. The decline in profits occurs both in the export and the manufacturing sectors. First of all a decline in the profit rate would diminish the expansion of the sector, and hence reduce to the economy the advantages of such expansion. Secondly, the savings, which arise mainly from the income from profits, would be reduced by a reduction in the profit rate. Savings being scarce and important, from the point of view of economic development, reductions in savings should be considered as losses. Thirdly, the fluctuations tend to reduce the size of the manufacturing sector in still another way. At low levels of income the losses to the firm may

be so large as to drive firms out of business, so that when income rises again only the more profitable firms would continue to operate. So that it is likely that the more violent the fluctuations the smaller would be the size of the sector.

Again, on the assumption, that the losses from the above causes are proportional to the extent that profits are reduced by fluctuations, we get the following expected loss from equations (3.62) and (3.65).

$$L_3 = v_6 \delta + v_7\left(1 - \frac{\ell_2 w}{\kappa_2}\right)(\delta_2 + \delta_3) \tag{3.76}$$

where v_6 and v_7 are constants.

(d) We saw that fluctuations tend to increase consumption, especially consumption of imported commodities. This increase in consumption reduces savings and foreign exchange earnings. Savings and foreign exchange play an important role in the process of economic development, and hence their reduction constitute losses from the point of view of development.

The decline of savings and foreign exchange, we saw, takes place at all levels of income, and their decline increases with increases in the variance of fluctuation in the export sector. Therefore, we shall assume that the losses of the above type are proportional to the variance of z. Hence we get the following loss function,

$$L_4 = v_8 \sigma^2 \tag{3.77}$$

where v_8 is a constant.

CHAPTER 4

ALLOCATION OF RESOURCES: CHOICE BETWEEN SECTORS

4.1. Introduction:

In the previous chapter we developed a model of an under-developed economy and discussed some of its implications. Next we discuss the problem of allocation of resources in such an economy.

The theory of allocation of resources plays an important role in economic thought, this is especially so in the case of the economics of under-developed countries. The correct allocation of resources being very significant for economic development and economic development being a major concern of these countries, this special interest in the theory of allocation seems quite natural. This is also reflected in the large proportion of the contemporary literature of the economics of under-developed countries devoted to the problems of allocation of resources. The theory of allocation poses some very interesting problems, and many of these problems, especially those in relation to under-developed countries, have not yet been solved to the satisfaction of everyone. We shall therefore, devote the first part of this chapter for the development of a logical basis for the analysis of allocation of resources in under-developed countries.

In many of the under-developed countries, the problem of allocation of resources have been left, so far, to the forces of the market mechanism. In section 4.2 of this chapter we shall first look at the conditions under which the market mechanism could be expected to allocate resources in a socially optimum manner, and then, see how well these conditions are satisfied in the under-developed countries. We shall

show that these conditions are not satisfied, and that therefore, the market forces cannot be expected to allocate the resources in a socially optimum manner. This implies the need for some direction of resources for it to be allocated in a socially optimum manner. A logical framework for such an investment policy is found in Tinbergen[1/] and Chenery.[2/] We shall extend this theory for the case of multiple aims, and use it as a logical basis for the critical evaluation of the different investment criteria[3/] that have suggested in the literature. We shall specially show that there is a very striking relation between this theory of investment policy and the 'social marginal product' criteria of Chenery.[4/] In the final section of this chapter, we analyse the problem of allocation between the export and the domestic manufacturing sectors of the model, as an application of the above theory of investment policy. The problem of allocation of resources within sectors we shall take up in the next two chapters.

4.2 Theory of Allocation and Economic Development.

4.2.1 The Perfectly Competitive Model. It can be shown, that the market mechanism will allocate the resources in a socially optimum manner if the

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1/ Tinbergen, J., On the Theory of Economic Policy, (Amsterdam: North-Holland Publishing Co., 1952); and Economic Policy, (Amsterdam: North-Holland Publishing Co., 1956).

2/ Chenery, H. B., 'Development Policy and Programmes', Economic Bulletin for Latin America, Vol. III, No. 1, March 1958, pp. 51-60.

3/ The term 'criterion' is used here as 'a rule of thumb' for allocation of resources; and its use is different in the literature of programming.

4/ Chenery, H. B., "application of Investment Criteria', Quarterly Journal of Economics, Vol. LXII, No. 1, (Feb. 1953), pp. 76-96.

perfectly competitive model is a good representation of the working of the economy. In a perfectly competitive model it is assumed that the product and the factor markets are competitive, that there is full freedom of entry of firms and factors, that there exists full knowledge of market conditions, that there are no external economies or diseconomies and that firms maximise profits and that each individual maximises his utility. In such a model, each firm motivated by maximum profits will undertake all investment for which the internal rate of return is greater than the interest rate, which is determined in a perfect capital market. Each individual acting to maximise his utility will supply labor in such quantities that the disutility of supplying the marginal unit is equal to the utility of the resulting income. Also each product will be consumed by each consumer in such a quantity that the resulting marginal utility will be worth giving up an income equal to its price. The resulting allocation under such conditions cannot make any individual better without making some one worse-off. Therefore, the resulting allocation from a perfectly competitive model is 'Pareto optimum'. The above result can be proved quite rigourously. It should be noted that this model is static, and that the distribution of income is assumed to be socially desirable.

Therefore, the extent to which the market mechanism can be expected to allocate the resources in a socially optimum manner depends on how closely the economy approximates a perfectly competitive model. That the under-developed countries deviate quite significantly from this model is now fairly widely recognised. Rosenstein-Rodan[5] was one of the

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[5] Rosenstein-Rodan, P. N., 'Problems of Industrialisation in Eastern and South-Eastern Europe', Economic Journal, (June-September 1953): also reprinted in Economics of Development, ed. Agarwala and Singh, op. cit., pp. 245-255.

earliest economists to focus attention on some of the difficulties of the market mechanism, in the allocation of resources in relation to the economic development of post-war eastern Europe. Among others who have more recently pointed to the ineffectiveness of the market mechanism in the allocation of resources in under-developed countries are Nurkse,[6] Myrdal[7] and Lewis.[8]

It is quite clear that the model considered in Chapter 3, is very far from a perfectly competitive model. In fact much of our emphasis in this model were on the imperfections. Even after the breaking up of the economy in to three fairly homogeneous sectors, we saw that the conditions of perfectly competitive model are not even satisfied within the sectors. The subsistence sector is perhaps the furthest from a perfectly competitive model; we saw that the basic economic organisation was feudal rather than that of a perfectly competitive economy. Very little organised production for the market or any investment takes place within this sector. In the export sector, though the economic organisation is not feudal, we saw that the ownership, and hence entry, was very restrictive. Also the long run nature of the investments makes the shifting of investments with the market conditions very difficult. Out of the three sectors, perhaps it is the domestic manufacturing sector that comes closest to a perfectly competitive model. Even here the deviation is quite significant,

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6/ Nurkse, R., Problems of Capital Formation in Under-developed Countries, (Oxford: Blackwell, 1953) pp. 4-24.

7/ Myrdal, G., Rich Land and Poor, (New York, Harper and Brothers, 1957) specially see, pp. 137-143.

8/ Lewis, W. A., The Theory of Economic Development, (Homewood, Illinois, Richard D. Irwin, Inc., 1955), pp. 90-101.

the ability to enter is limited, the capital market is rather imperfect and the knowledge of market conditions is far from desirable. The imperfections in the market conditions are important to the understanding of these economies, and a model that abstracts from these imperfections would therefore exclude an important characteristic of these economies.

Apart from the imperfections in the market conditions, there are other reasons for the inapplicability of the perfectly competitive allocation model to under-developed countries. Many writers have drawn attention to the importance of external economies in the allocation of resources in under-developed countries, and that these should be recognised in the proper allocation of resources under-developed countries. As we noted before the above model abstracts from external economies. A further weakness of this model is that it is a static model. The dynamic considerations of allocation become specially important, since economic growth is a major aim of the under-developed countries. Still a further weakness in the perfectly competitive model in the allocation of resources in the under-developed countries is the implicit assumption that the distribution of income before and after allocation is desirable by society. The disparity of income in the under-developed countries is well known, and as Chenery remarks, "the tendency has been for the inequalities to increase."[9]

The best illustration of the failure of the market mechanism in the proper allocation of resources of under-developed countries, is perhaps, the historical patterns of growth of these countries. Historically, the process of allocation of most of the under-developed countries,

9/ Chenery, H. B.,'Development Policies and Programmes', op. cit., p. 53.

in some continued to the present day, have been based on decentralised investment decisions guided by profit incentives. It is well known that past patterns of investments, are in no small way responsible to the present economic ills of these countries.

4.2.2 A Logical Basis for the Allocation of Resources in Underdeveloped Countries. What follows from the above consideration is that the perfectly competitive model of allocation is not appropriate for under-developed countries; and therefore, the market mechanism cannot be expected to allocate the resources of these countries in a socially optimum manner. The failure of the market mechanism calls for the direction of investment in some socially optimum manner. It further calls for a theory of allocation which would help guide this direction of investment; i.e., a theory of investment policy. We next discuss a logical basis for such an investment policy.

There have been many attempts towards the derivation of a 'rule of thumb', or what has been called an investment criterion, for the proper allocation of resources in under-developed countries, when the market mechanism is known to have failed. It is very unlikely that such a criterion would be satisfactory under all conditions, though many economists have tried to obtain such a general investment criterion. It would depend on the aims of the society and the relative abundance of resources. In much of the literature on investment criteria, it is not clear what the aims of these criteria are, and under what conditions the criteria hope to achieve these aims. Much of the controversy on the investment criteria would have been avoided had these conditions under which the different criteria are optimum had been explicitly

stated. It is possible to obtain a logical framework, by the extension of the theoretical concepts of Tinbergen and Chenery on investment policy, for the systematic analysis of these different investment criteria. This will be our next consideration.

First, we shall define some of the concepts used by Tinbergen.[10] Decisions on allocation are assumed to be made at two levels. The decision making body at the higher level we shall call the '<u>policy maker</u>'; this could be a single institution like the legislature of the country, or a group of institutions consisting, say of labor unions, management and representatives of the legislature. The decisions at the next level we shall call the '<u>planning authority</u>'; this would be a technical organisation who would be carrying out the decisions of the 'policy makers'. The 'policy maker' first defines a set of '<u>aims</u>' of society,[11] which would be a qualitative or quantitative; aims that the society wishes to achieve. In general these aims would be multiple. Chenery gives the following as commonly taken 'aims';[12]

(i) Maximisation of income or per capita income,

(ii) Improved distribution of income by regions,

(iii) Reduction of risk of fluctuations of income due to crop failure and variations in export markets,

(iv) Maximisation of employment.

10/ Tinbergen, J., <u>Economic Policy</u>, <u>op. cit</u>.

11/ The present analysis assumes that the aims of society are given as datum. However it is recognised that the social scientist has a role in determining these aims.

12/ Chenery, H. B., <u>op. cit</u>., p. 54.

In the present analysis we shall restrict ourselves to those aims that can be quantified and also assume that it is the aim of the 'policy maker' to maximise (or minimise) each of these aims. The variables with which the policy maker wishes to attain the aims are called the 'instrumental variables', these could vary from tax rates and foreign exchange control to the direct control of the investments.[13] In our analysis we shall restrict to a single instrumental variable; i.e., each of the aims is assumed to be a function of a single variable.

The purpose of the 'policy maker' is to realise the 'best' future configuration of the stated 'aims' of society. Here we are only concerned with investment policy. Investment policy, however, will not be completely independent of economic policy in general - the optimum investment decisions would depend on fiscal policy, monetary policy and foreign trade policy. For the present purpose we shall abstract from these considerations.

If there were only a single aim, the attainment of the optimum investment policy would be rather simple; the maximisation of this single aim under the restrictions of available resources. But in the case of multiple aims, the choice of the optimum investment policy would depend on the manne- in which the 'policy maker' weights the alternative aims.

We shall assume that given any two different combinations of the aims, the policy maker is able to prefer one to the other, or remain

[13] Theil, H., on 'Econometric Models and Welfare Maximisation', Weltwirtschaftliches Archives, Band 72, Heft 1, (1954), pp. 60-83, considers two sets of variables; instrumental and indirect. The latter being defined as 'those that are affected indirectly by the use of indirect variables'. This differenciation is very similar to the difference between endogenous and and exogenous variables in Econometric models.

indifferent between them. It can be shown, in a manner similar to the derivation of the utility function of the theory of consumer behavior, that if these preferences satisfies the further assumptions of irreflexivity, transitivity and continuity, there exists a 'welfare function' of the 'policy maker' of the form,

$$W(\Lambda_1, \Lambda_2, \dots, \Lambda_n) \tag{4.1}$$

(where $\Lambda_1, \Lambda_2, \dots, \Lambda_n$, correspond to the different aims) such that $W(\Lambda^1) > W(\Lambda^2)$ if and only if Λ^1 is preferred to Λ^2, where Λ^i corresponds to the vector $\{\Lambda^i_1, \Lambda^i_2, \dots, \Lambda^i_n\}$; $i = 1, 2$.

If this 'welfare function' is available to the 'planning authority' in an explicit form, the problem of obtaining the optimum investment policy would be the same as if there were only a single aim, i.e., the maximisation of this known 'welfare function'. But, usually such a welfare function is not explicitly available to the planning authority. The question that we shall next ask is; what is his best decision when all that is available to him are the 'aims' of the 'policy maker' and not the explicit form of his 'welfare function'?

The 'planning authority' could obtain a subset of the set of all feasible decision, such that the optimum decision of the 'policy maker' would be a member of this subset for all possible 'welfare functions'. This would be the smallest subset of feasible decisions, such that, to every feasible decision outside this subset, there will be a decision within the subset which would be at least as 'good'. We shall refer to such a subset of decisions as a 'Pareto optimum' set. If the explicit

form of the 'welfare function' of the 'policy maker' were not available to the 'planning authority', the most the latter could do would be to obtain the 'Pareto optimum' set of decisions. The above logical framework could be a basis for the appraisal of the many investment criteria suggested in the literature. These investment criteria suggest 'rules of thumb' to be followed in the allocation of resources in under-developed countries. In much of this literature it is not clear what 'aims' the proposed criteria hopes to achieve. This is specially important in comparing two different criteria, for in such cases if the aims of the criteria were different, there would be no basis for the comparison of the criteria. Once the 'aims' of the criteria are made clear, it is then possible to determine whether the suggested criteria maximises the 'aims' subject to the restrictions of the problem.

The 'social marginal product criteria' of Chenery[14] has a close resemblance to the above formulation, and to the rest of our analysis of this chapter.

Chenery expanded the concept of 'social marginal product' in resource allocation suggested earlier by Kahn.[15] Kahn had argued that it is not the private marginal product that should assign priorities in investment, but the social marginal product; i.e., the total contribution to the product from the point of view of the whole society. He specially emphasised that the factor costs that should matter under such conditions are the opportunity costs.

[14] Chenery, H. B., 'Applications of Investment Criteria', op. cit.

[15] Kahn, A., 'Investment Criteria in Economic Development Programs', Quarterly Journal of Economics, (Feb. 1951), pp. 38-61.

Chenery introduced a 'social welfare function' of the following form as a desired aim of investment policy:

$$W(Y, B, D, \ldots,) \tag{4.2}$$

where Y denotes the total national income, B the balance of payments, and D the distribution of income.

From this equation, we get that the effect on 'social welfare' of a increment of investment is,

$$\Delta W = \frac{\partial W}{\partial Y} \Delta Y + \frac{\partial W}{\partial B} \Delta B + \frac{\partial W}{\partial D} \Delta D + , \ldots , \tag{4.3}$$

which would constitute the social marginal product of the investment. Restricting to two variables, the national income and the balance of payments, he uses the following form of the 'social marginal product',

$$S \cdot M \cdot P = \Delta W = \Delta Y - r \, \Delta B \tag{4.4}$$

where ΔY is the total net effect on the national income, ΔB the net effect on the balance of payments on the investments, and $r = \frac{\partial W}{\partial B}$ is interpreted as 'equivalent of the national income of a unit increase in the balance of payments'. Chenery uses the above formulation of the 'social marginal product' for the comparison of investment projects of six Mediterranean countries.

In the above formulation it is implicitly assumed that the social welfare function or the way in which society weights the different 'aims'

is known in an explicit manner. It is assumed for instance that the coefficient r is known explicitly. If this were so, the above procedure would be approriate for granting investment priorities. But as we have pointed out, it is not always that the social welfare function is known in an explicit manner.

The Chenery criteria, however, is very similar to the formulation given at the beginning of this section. The variables in the above 'welfare function' can be interpreted as what we called the 'aims' of the 'policy maker'. The Chenery 'welfare function' itself to what we called the 'welfare function' of the 'policy maker', except that the latter function was not assumed to be known in an explicit manner.

4.3 Choice Between Sectors.

In this section we shall discuss the optimum procedure for the allocation of resources between the export and the domestic sector of the economy.

A major characteristic of the type of economy that we are analysing is the rather violent fluctuations in its income. As we discussed in the previous chapter, these fluctuations give rise to many losses, and are a major weakness in the smooth working of these economies. It is generally recognised both by the economists and the political thinkers of these countries, that for the stimulation of the process of development of these countries, and for the general welfare of its people, that these fluctuations should be checked. As mentioned in Chapter 1, the General Assembly of the United Nations appointed a group of economists in 1953 to study the problem of the fluctuations in the prices of primary

commodities and recommend methods for the stabilisation of these prices.[16] It seems quite natural, therefore, that an 'aim' of investment policy of the primary producing countries be the minimisation of the fluctuations in the income of these economies.

As a measure of the extent of these fluctuations, we shall take the variance of the distribution of income of the export sector of the economy.[17] The fluctuations in the income of the other sectors of the economy, we saw in the last chapter, were induced by the fluctuations in the export sector of the economy, so that they can be said to be functionally related to the extent of fluctuations in the export sector. Therefore, the variance of income distribution of the export sector is an appropriate variable for the measure of the extent of fluctuations of the income of the whole economy.

As the on other 'aim' of the investment policy we take the maximisation of the expected value of the total income of the economy. Given a stationary distribution of income, the expected value can be interpreted as the average long run income of the future. So that the 'aim' is not a short run 'aim', but a long run average, and except for the fact that it weighs the future the same as the present, seems quite an appropriate 'aim'. However, it must be mentioned that this does not necessarily maximise the rate of growth of the economy. The introduction of an 'aim', to maximise growth like the aim of the Galenson

[16] United Nations, Commodity Trade and Economic Development, op. cit.

[17] See Footnote 5 of Chapter 3.

and Leibenstein[18] criteria, would highly complicate our analysis. Thus though we fully recognise this to be a very appropriate 'aim' of economic policy of an under-developed country, we shall not introduce it to the present analysis.

Therefore, for investment policy of our model, we shall consider the 'aims' as: (1) minimisation of the variance of the export sector, and (2) maximisation of the income of the export sector.

The 'social welfare function' of the policy maker takes the form,

$$W(Y, V) \tag{4.4}$$

where Y denotes the total expected income, and V the variance of the distribution of income of the export sector, a measure of the extent of the fluctuations of the income of the economy. This function is not assumed to be known explicitly.

There are three main factors of production in our analysis, labor, land and capital. As we have assumed throughout our analysis, labor is not a restricting factor of production. Since land also enters in constant ratio to capital, we need consider only one restricting factor of production, capital. Therefore, we shall assume that the only restriction arises from the total available capital stock.

First, we shall discuss the allocation of <u>total</u> capital between the two sectors of the economy. Next, we take the problem of <u>marginal</u>

[18] The 'aim', though not quite explicitly stated, of their criteria (Galenson, W., and H. Leibenstein, 'Investment Criteria, Productivity and Economic Development', <u>Quarterly Journal of Economics</u>, Vol. LXIX, No. 3, (August 1955), pp. 343-370) is the maximisation of the growth rate.

investment of capital. The optimum allocation of marginal investment is very much dependent on the already existing pattern of the distribution of investment between the two sectors.

The income of the export sector, from equation 3.5 of Chapter 3, is,

$$(3.5) \qquad Y_1 = f(L)\, z$$

where z is a random variable, L the land under cultivation. The capital land ratio is assumed constant, therefore, the above equation can be written in the form,

$$Y_1 = f(\kappa_1 K_1)\, z$$

In Chapter 3, we assumed that for low enough values of z, the maximisation of profits in the firms of this sector resulted in the cultivation of only part of the land, so that for certain low values of z the land under cultivation was a function of z. Here, we shall assume, that all the land is cultivated for all values of z. Or that the industry will work at full capacity for all values of z.

Therefore, the expected income from the export sector is,

$$(4.5) \qquad E(Y_1) = f(\kappa_1 \bar{K}_1)\, E(z)$$

and the variance is,

$$(4.6) \qquad V = [f(\kappa_1 K_1)]^2\, \sigma^2$$

The income of the domestic manufacturing sector, is from equation 3.17,

$$X_2 = \kappa_2 K_2 \tag{3.17}$$

where κ_2 is a constant, K_2 the capital stock, and X_2 the output at full capacity.

The demand d_2 for the product of this sector is given by equation (3.48) of Chapter 3,

$$\begin{aligned} d_2 &= b_{20} + b_{21} L + b_{22} f(L) z , b_{23} X_2 \\ &= b_{20} + b_{21} \kappa_1 \bar{K}_1 + b_{22} f(\kappa_1 \bar{K}_1) z + b_{23} X_2 \end{aligned} \tag{4.7}$$

We assumed that the output was equal to $\bar{X}_2$ if the demand for the product was greater than d_2, if the demand was less than the industry would produce to meet this demand; i.e., in this case the industry would be producing at less than full capacity. i.e.,

$$\begin{aligned} Y_2 = \bar{X}_2 = \kappa_2 K_2 \quad & \text{if} \quad d_2 > \bar{X}_2 \\ = d_2 \quad & \text{if} \quad d_2 < \bar{X}_2 \end{aligned}$$

Let $\bar{z}$ be the value of the random variable z for which i.e.,

$$\begin{aligned} \bar{X}_2 &= d_2 \\ \bar{z} &= \frac{1}{b_{22} f(\kappa_1 K_1)} [\bar{X}_2(1 - b_{23}) - b_{20} - b_{21} \kappa_1 K_1] \end{aligned} \tag{4.8}$$

For $z > \bar{z}$, the industry would be working at full capacity. Let

$$v = d_2 - \bar{X}_2 \qquad z > \bar{z} \tag{4.9}$$

represents the extent of excess demand in the industry. For any value of the random variable z this is a function of K_1 and K_2. Excess demand would mean that these commodities would now have to be imported, resulting in an extra drain on foreign exchange. Therefore, we shall assume that excess demand results in losses to the economy, and let this loss function be of the form,

$$\xi(v) \tag{4.10}$$

Similarly, for values of $z < \bar{z}$ the industry would be working at less than full capacity. The extent of under capacity of the industry in this case is,

$$s = \bar{X}_2 - d_2 \qquad z < \bar{z} \tag{4.11}$$

This again is a function of the capital stock of the two sectors, K_1 and K_2. The working of the industry at less than full capacity would also result in losses to the economy; factors would now be under-utilised, the displaced labor has to be in some way supported by society, and it is likely that the plant has to be maintained. So that they can be reutilised without loss of efficiency when demand returns to normal. Let this be represented by a loss function of the form,

$$\eta(s) \tag{4.12}$$

The equation connecting d_2, $\bar{X}_2$, v, and s, can be represented in the form

$$d_2 = \bar{X}_2 + v - s \tag{4.13}$$

where

$$v \geq 0, \qquad s \geq 0$$

also we have,

$$\text{if} \quad v > 0 \quad \text{then} \quad s = 0$$

and

$$s > 0 \quad \text{then} \quad v = 0$$

The net gain, from this sector when d_2 is the demand can be represented in the form,

$$\begin{aligned} \Phi(\bar{K}_2 | d_2) &= \bar{K}_2 - \xi(v) \qquad & z > \bar{z} \\ &= d - \eta(s) \qquad & z < \bar{z} \end{aligned} \tag{4.14}$$

The above function is dependent on the capital stock of both the sectors of the economy, K_1 and K_2. Also, since z is the random variable

responsible for the variation the above function can be represented in the form,

$$\Phi(K_1, K_2 | z) \tag{4.15}$$

Now, if $-\xi(v)$ and $-\eta(s)$ are concave in K_1 and K_2, then the above function is concave in K_1 and K_2.

We shall use the following theorem due to Danzig[19] to prove the concavity of the total expected gain.

<u>Theorem</u>: If $\Phi(X_1 \cdots X_n | \theta)$ is a concave function over a fixed region Ω for every value of θ, then any positive linear combination of such a function is also concave in Ω.

From the above theorem it follows, that since $\Phi(K_1, K_2 | z)$ is concave for all z, the expected value of this function,

$$E_z\{\Phi(K_1, K_2 | z)\} \tag{4.16}$$

is concave in K_1 and K_2.

Also, because of our assumption of diminishing returns for the production function of the export sector, $f(\kappa_1 K_1)$ is a concave function in K_1.

The total expected income,

$$\Phi(K_1 K_2) = E\,\Phi(K_1 K_2 | z) + f(\kappa_1 K_1)\,E(z) \tag{4.17}$$

[19] Danzig, G. B., 'Linear Programming Under-uncertainity', <u>Management Science</u>, Vol. 1, (1955), p. 200.

which is the sum of two concave functions in K_1 and K_2 is itself a concave function in K_1 and K_2.

We shall next show that if $\bar{\Phi}(K_1 K_2)$ is concave for all K_1 and K_2, and if it is an increasing function of K_1 and K_2, then the equi-income curves are convex to the origin.

For, if $\bar{\Phi}(K_1, K_2)$ is concave, then for any $K^1 = (K_1^1, K_2^1)$ and $K^2 = (K_1^2, K_2^2)$ we have,

$$\lambda \bar{\Phi}(K^1) + (1 - \lambda) \bar{\Phi}(K^2) \leq \bar{\Phi}(\lambda K^1 + (1 - \lambda) K^2)$$

$$\text{for } 0 \leq \lambda \leq 1$$

Now, if the expected gain for any K^1 and K^2 are the same, we have,

$$\bar{\Phi}(K^1) = \bar{\Phi}(K^2) = \bar{\Phi}_0$$

Therefore,

$$\bar{\Phi}_0 \leq \bar{\Phi}(\lambda K^1 + (1 - \lambda) K^2)$$

Thus as in Figure 1, if the equi-income curves are increasing functions of capital stocks K_1 and K_2, the equi-income curves are convex.

We now consider the problem of allocation of a given stock of capital $\bar{K}$ between the two sectors of the economy.

The allocation problem can be formally stated as follows,

$$\text{(4.18)} \qquad \max. \quad Y = \bar{\Phi}(K_1 K_2)$$

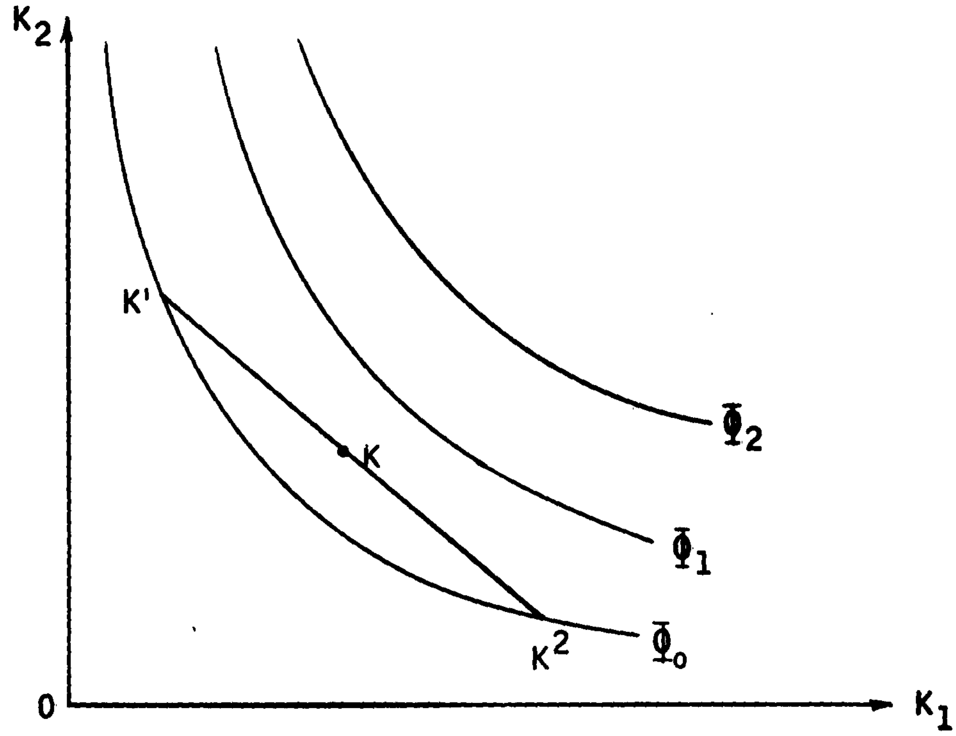

Figure 1.

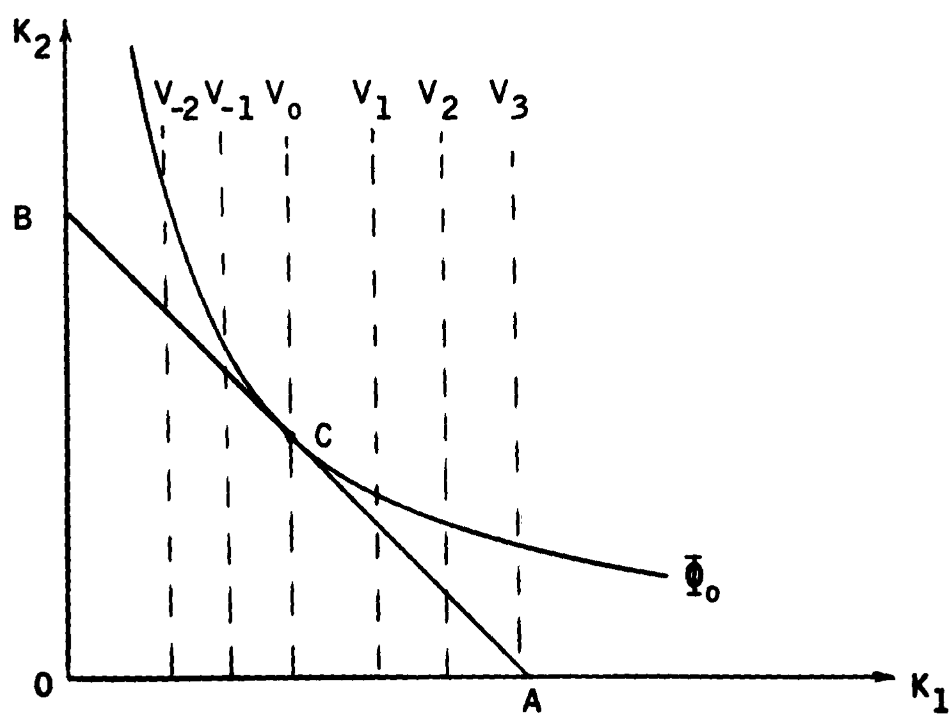

Figure 2.

(4.19) $$\min. \quad V = [f(\kappa_1 K_1)]^2 \cdot \sigma^2$$

(4.20) $$\text{subject to} \quad K_1 + K_2 \leq \bar{K}$$

The allocation problem is one with multiple 'aims'. Thus, as discussed in Section 4.2.2 no single 'optimum' solution can be obtained without the knowledge of the preferences of the 'policy maker'. Since we are not assuming any explicit form of the 'welfare function', we shall have to confine our discussion to 'Pareto optimum' set of solutions.

Let the Line AB in Figure 2 represent the capital restriction (4.20). Then OAB is the feasible set. Also, let Φ_0 be the maximum attainable total gain under the restriction (4.20), and C the point at which this income is reached. Then the equi-income curve will be tangential to AB at C, further at C we have,

(4.21) $$\left.\frac{\partial\bar{\Phi}}{\partial K_1}\right|_c = \left.\frac{\partial\bar{\Phi}}{\partial K_2}\right|_c = \lambda$$

The second 'aim' (4.19) depends only on the single variable K_1. It is also an increasing function of K_1. Thus equi-variance lines will be parallel to the OK_2 axis and increase with K_1.

We shall next show that the line segment BC of Figure 3, is the 'Pareto optimum' allocation set for capital endowment K.

For any point in the interior of OAB, consider the point at which the line through it parallel to OB intersects AB. This latter point will have the same variance but a higher income compared to the former

point. Therefore, for every point on the interior of OAB, there is a more preferred point on AB. Also, because of the convexity of the equi-income curves, the equi-income curve passing through every point on AC will intersect BC or OB. Therefore, to every point on AC, there is a point on OBC which has the same income but lower variance, hence points preferred to points on AC. But since B is preferred to all points on OB, the line BC represents the 'Pareto optimum' set for the above problem.

Let OCC' in Figure 3 be the locus of points of maximum income for different endowments of capital. We shall now show that this locus OCC' is convex.

At all points of OCC', from equation (4.21) we have,

$$\frac{\partial\bar{\Phi}}{\partial K_1} = \frac{\partial\bar{\Phi}}{\partial K_2}$$

Now because of our assumption of diminishing returns to scale in the export sector, the marginal returns to K_1 decrease with every increase in K_1. However, the assumptions for the domestic manufacturing sector makes its returns to scale constant. Though $\frac{\partial\bar{\Phi}}{\partial K_2}$ is a function of both K_1 and K_2, the influence of K_1 on this function is very small. Thus the marginal returns to $\frac{\partial\bar{\Phi}}{\partial K_1}$ would decrease faster than the marginal returns to $\frac{\partial\bar{\Phi}}{\partial K_2}$. Therefore, we have,

$$(4.22) \qquad \frac{\partial^2\bar{\Phi}}{\partial K_1} < \frac{\partial^2\bar{\Phi}}{\partial K_2}$$

Therefore, the locus OCC' is convex. It also follows from this that

as we proceed along this locus the proportion of income in the domestic manufacturing sector would increase.

The investment decisions that arise in practice is not the allocation of the total resources but the allocation of marginal investment given the historical distribution of investment. Due to the long run nature of the investments, the investment allocated in the past cannot be changed, at least in the short run. We shall next consider the problem of allocation of marginal income.

In the demand equation for the products of the domestic manufacturing sector (3.48), we considered the demand to vary only with income. The possibility of import substitution or the possibility of creating new demand is not considered. The allocation decision would depend very much on whether the new investment is to go to the increase of the capacity of the old industries or for import substitution. We first consider the problem of allocation of marginal investment when the demand function is as in (3.48), next the possibility of the shift in the demand function with import substitution is considered.

Let B'C'A' in Figure 3 be the capital restriction corresponding to a total endowment of $K + \Delta K$, or BCAA'C'B' is the increase of the feasible set corresponding to an incremental increase ΔK of capital. We know that B'C' is the new 'Pareto optimum' set. The position that could be reached on B'C' would depend on the historically given position on BCA. If all of the capital stock $\bar{K}$ had been previously allocated, the historical distribution would be on some point of ABC, not necessarily on BC. If this position is on H_1A of AB, then it is clear that the whole of the increment in investment will be allocated

into the domestic manufacturing sector, i.e., $\Delta K = K_2$. But if the historically given position is on BH_1, there will be more than one possible distribution of investment allocation. The choice of the 'optimum' distribution would depend on the 'welfare function' of the 'policy maker'. For points closer to C' the total expected income is high, but so also is the variance. For example if the historically given position is H_2, the policy maker will have a choice in the region $J_1 J_2$.

Next consider the possibility of a shift in the demand function. Such a shift could arise in many ways. It could arise, for instance, from a new ability to produce domestically a commodity which has so far been imported, or by the substitution of a domestically produced commodity for a imported good. Before considering the allocation policy, we shall consider the effect of the shift in the demand function on the 'Pareto optimum' .et.

A positive shift in the demand function will increase the marginal gain of an investment in K_2. i.e.,

$$\frac{\partial \bar{\Phi}^1}{\partial K_2} > \frac{\partial \bar{\Phi}}{\partial K_2} \tag{4.23}$$

where $\bar{\Phi}^1$ is the total gain function with the shift in demand, and as before $\bar{\Phi}$ is the total gain function without a shift in demand.

As before, we have from equation (4.21), the relation

$$\frac{\partial \bar{\Phi}}{\partial K_2} = \frac{\partial \bar{\Phi}}{\partial K_2} = \lambda$$

The shift in the demand function for the domestically manufactured products, does not change the marginal returns to capital of the export sector, also $\frac{\partial\bar{\Phi}}{\partial K_1}$ primarily dependent on the marginal returns to capital of the export sector. Therefore, we have,

(4.24) $$\frac{\partial\bar{\Phi}^1}{\partial K_1} = \frac{\partial\bar{\Phi}}{\partial K_1}$$

From (4.23), (4.21) and (4.24) we get,

(4.25) $$\left.\frac{\partial\bar{\Phi}^1}{\partial K_2}\right|_C > \left.\frac{\partial\bar{\Phi}^1}{\partial K_1}\right|_C$$

Thus, the substitution of K_2 for K_1 would increase the net return. If D is the new point of maximum gain, then as in (4.21) it must satisfy the relation,

(4.26) $$\left.\frac{\partial\bar{\Phi}^1}{\partial K_1}\right|_D = \left.\frac{\partial\bar{\Phi}^1}{\partial K_2}\right|_D$$

Therefore, D must lie on the left of C.

Let OD be the locus of maximum gain with the shift in the demand function.

If $\bar{\Phi}_o^1$ is the maximum gain that is attainable with the shift in the demand function, then in general,

(4.27) $$\bar{\Phi}_o^1 > \bar{\Phi}_o$$

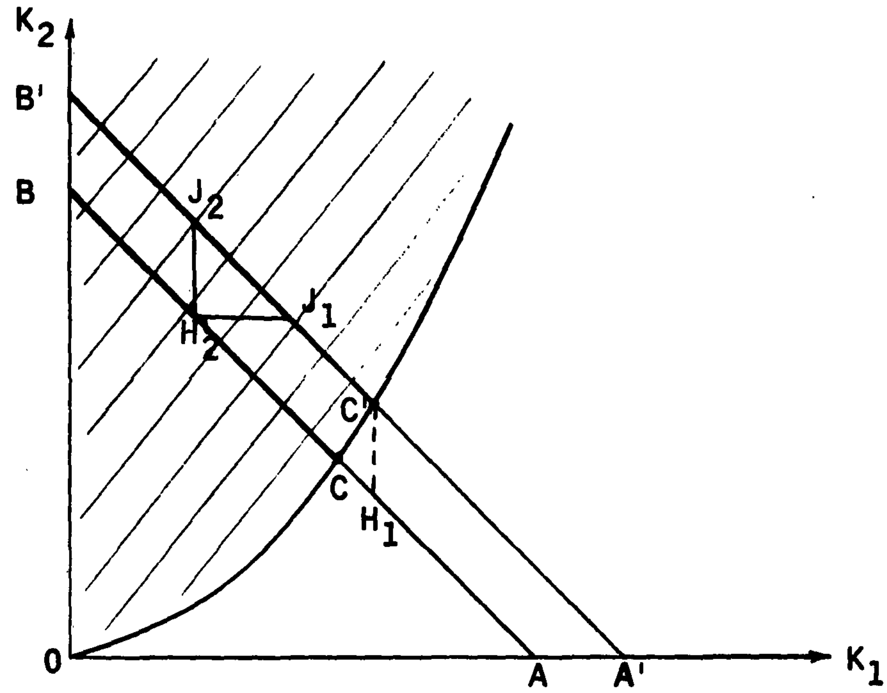

Figure 3.

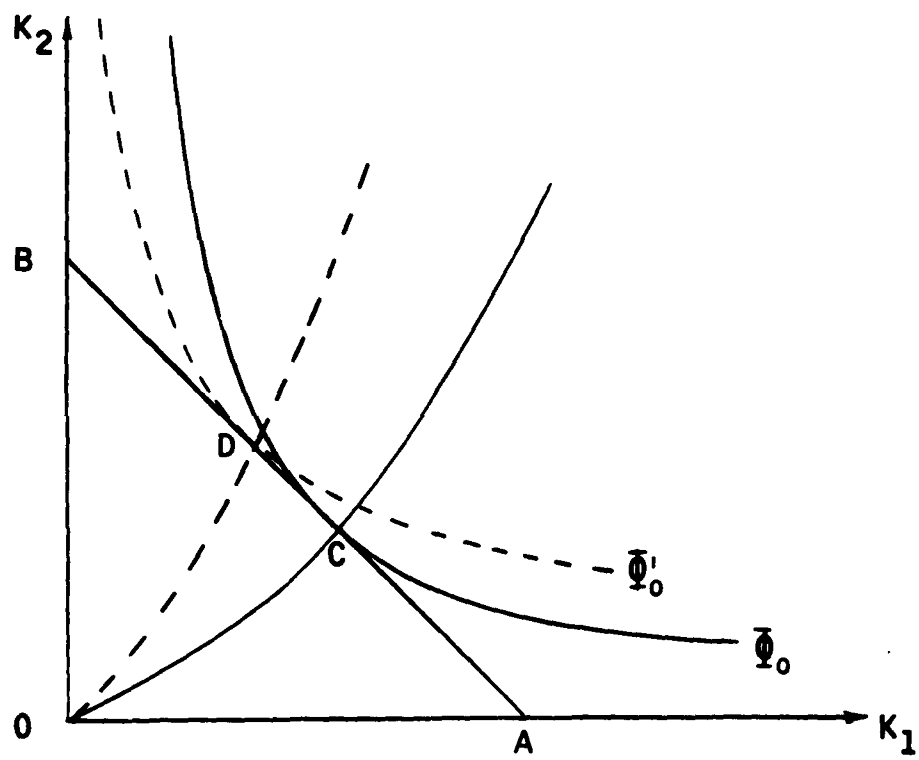

Figure 4.

This can be seen in the following manner. If $\bar{\Phi}^1_C$ is the new equi-profit curve at C, then in general $\bar{\Phi}^1_C \geq \bar{\Phi}_o$.

For, at this point the returns from the export sector would be the same $\bar{\Phi}$ and $\bar{\Phi}^1$ and in general the returns from the domestic manufacturing sector would be greater for $\bar{\Phi}^1$ with a positive shift of the demand function. Further,

$$\bar{\Phi}^1_o > \bar{\Phi}^1_C$$

Since Φ^1_o is assumed to be the maximum. Therefore, we have,

$$\bar{\Phi}^1_o > \ddot{\Phi}_o$$

Thus the introduction of import substitution has three effects on the 'Pareto optimum' set. (a) The new 'Pareto optimum' set is a subset of the original set. (b) The maximum variance is reduced, (c) The maximum gain is now higher.

Finally we consider the investment decision when the increment in investment could go into import substitution, or in general would result in a shift of the demand function of the domestic manufacturing sector.

In Figure 5, let A'D'B' be the capital restriction, corresponding to an endowment of $\bar{K} + \Delta K$, and let B'D' be the new 'Pareto optimum' set. Thus if the historically given allocation is in the region AH_3 the whole of the increment of investment would go into the domestic manufacturing sector. But if the historical distribution is in H_3B, then there will be more than one allocation possibility depending upon the preferences of the policy maker.

The important point to note in this case is that even from a 'Pareto optimum' position as from a point on CH_3 of Figure 5 the optimum decision with import substitution is the allocation of the total increment of investment to the domestic manufacturing sector.

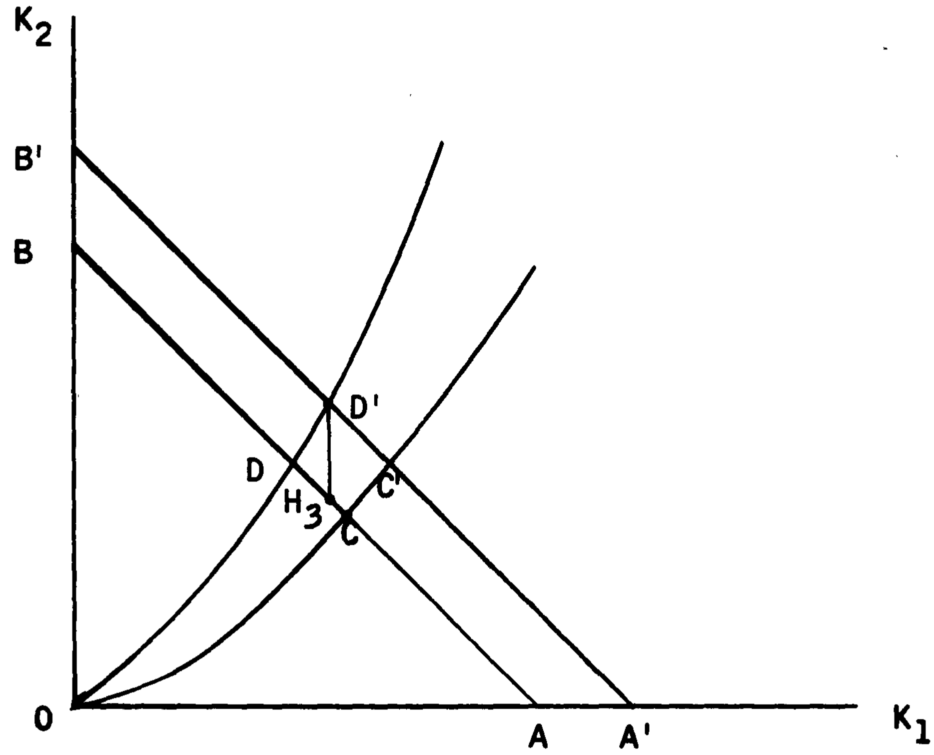

Figure 5.

CHAPTER 5

ALLOCATION OF RESOURCES WITHIN SECTORS: (i) EXPORT SECTOR

5.1 Introduction.

In the next two chapters we shall consider the problem of allocation of resources among different industries of the export and the domestic manufacturing sectors of the economy. In the solution of these two problem we resort to the techniques of programming under uncertainty. The problem of allocation within each of these sectors gives rise to an interesting problem in programming under uncertainty. In the export sector the uncertain elements, as we have already seen, arise from the output and the prices of the products of this sector. While in the case of the domestic manufacturing sectors, they arise from the random demand for the products of that sector.

Programming under uncertainty is in itself a new and interesting field of study. A satisfactory treatment of the general problem of programming under uncertainty is not yet available in the literature. The only such attempt, as far as we know, at such an exposition is by Madansky,[1] though this is quite original in many respects, is not altogether satisfactory as a general treatment of the subject. We shall, therefore, briefly discuss in the next section some of the problems that arise in programming under uncertainty, specially those in relation to the main problem of our present study, and suggest methods for their

[1] Madansky, A., Some Results and Problems in Stochastic Linear Programming, The RAND Corporation, Research Papers P-1598, January 19, 1959.

solution. The next part of this chapter is devoted to the discussion of the problem of allocation of resources between industries of the export sector.

5.2 Programming Under Uncertainty.

The general mathematical programming problem can be stated as the determination of an activity vector $x = \{x_1, x_2, \dots, x_r\}$, such that

$$(5.1) \qquad \max. \quad f(x_1, x_2, \dots, x_r)$$

$$(5.2) \qquad \text{subject to,} \quad g_k(x_1 \cdots x_r) \leq c_k \quad k = 1, 2, \dots, n$$

$$\text{and} \quad x_i \geq 0$$

where $f, g_1, \dots, g_n$ are functions defined on $x_1, x_2, \dots, x_r$, and $c_1, \dots, c_n$ are constants.

The programming problem under uncertainty could be said to arise when some or all of the parameters of (5.1) or (5.2) are random variables, i.e., if any of the coefficients of the functions, $f, g_1, \dots, g_n$ or if any of the constants $c_1, \dots, c_n$ are random variables.

In the recent literature, there are quite a few papers dealing with programming problems under uncertainty.[2] Many of these problems arise

[2] Danzig, G. B., "Linear Programming Under Uncertainty", Management Science, Vol. 1, (1955), pp. 197-206; Elmagraby, S., "Programming Under Uncertainty", Ph.D. Thesis, Cornell University, 1958; Markovitz, H., "Optimisation of Quadratic Functions with Respect to Linear Constraints", Naval Research Logistic Quarterly, Vol. 3, (1956), pp. 111-133; also by the same author, Portforlio Selections, Cowles Commission Monograph 16, (New York: John Wiley and Sons, Inc., 1959) Chapters 7, 8 and 9; Madansky, A., op. cit. and Inequalities for Stochastic Programming, The RAND Corporation, Research Papers P-1600, November 19, 1958.

from interesting and important practical problems, which points to the importance of programming under uncertainty as a tool in applied research.

Madansky in his analysis of the general programming problem under uncertainty, differentiates between two groups of problems: (i) where the outcome from the random variables are known before the choice of the optimum activity vector, (ii) where the choice of the activity vector has to be made before any knowledge of the outcome of the random variables. The first is a solution 'ex-post' while the second is a 'ex-ante' solution with respect to the random variables. We shall, therefore, call these the 'ex-post' and the 'ex-ante' problems respectively.[3/] This breakdown is a useful one, specially in relation to some inequalities derived by Madansky.[4/]

The method of solution of the 'ex-post' problem is the same as the methods of solution of the programming problem under certainty. However, there is a probabilistic interest in this problem too; namely, the analysis of the distribution of this solution for different values of the random variables, further, this is related to the 'ex-ante' solution by the Madansky inequalities.

However, the more interesting is the 'ex-ante' problem. The determination of solutions to this problem raises the more general question of decision under uncertainty. Decision under uncertainty has been widely treated in the literature, and therefore, we need not go into it

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3/ Madansky uses the terms 'wait-and-see' and 'here-and-now' to denote these two types of problems, however, we prefer to use the above terms 'ex-post' and 'ex-ante'.

4/ Inequalities for Stochastic Programming, op. cit.

in detail here. It should be noted that the solution of such a problem would be optimum only in the 'ex-ante' sense, and not necessarily optimum 'ex-post', once the values of random variables are known. Programming under uncertainty also raises the possibility of the optimum 'ex-ante' activity vector being non-feasible.

Madansky[5/] in his analysis lays rather strong emphasis on the feasibility of the optimum activity vector. It seems to us, however, that the question of feasibility should be incorporated within the general solution. When the coefficients of the restriction $f, g_1, \dots, g_n$ or of the coefficients $c_1, \dots, c_n$ are random variables, the feasibility of the optimum set of activities will not be certain. Thus under such conditions the feasibility of the optimum 'ex-ante' solution would arise. The approach suggested by Madansky, namely the restriction to only those activities, the probability of whose feasibility is greater than a given value P, does not seem to be quite appropriate. For, firstly, unless $P = 1$, this does not completely eliminate the non-feasibility of the optimum 'ex-ante' solution. Secondly, this classifies the activities into two groups, and this implicitly assumes that the consequences of a choice of an activity within either of these groups is the same. It is very likely that the suitability of an activity would depend on the extent of the non-feasibility of the activity vector. For example, in the problem that we would be discussing in the next chapter, the constant c's of the restrictions would become the random demand, and the activity vector would be represented by planned output, hence

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5/ Some Results and Problems in Stochastic Linear Programming, op. cit., examples I, II, and III on pages 2 and 3.

the 'ex-ante' optimum output vector need not be feasible. Non-feasibility in this case means the production of output in excess of demand, and the consequence of such excess production would depend on the extent of such over production. Thus the consequences would not be the same for all non-feasible activity vectors.

We next consider another approach to the 'ex-ante' problem. Consider the real valued function $\Lambda(x, z)$ for every possible activity x and every possible outcome of the random variable z. The function Λ can be interpreted as 'pay off' function in a game against nature. Following our terminology in Section 4.2.2 of Chapter 4, this function could be called the 'aim' of the programming problem. Or, Λ represents the consequence of the selection of the activity x and z the outcome of the random variable. So that the 'best' solution to the above problem is to select an activity vector x, which maximises (or minimises) $\Lambda(x, z)$.

It is quite possible, that there would be not a single such function, but a vector of such functions; such that the betterment of one of these functions does not necessarily lead to the betterment of the other. In Section 4.2.2 of Chapter 4, we had in connection of investment policy of under developed countries, a case of conflicting multiple 'aims', as the maximisation of employment, maximisation of employment and the reduction of the extent of the fluctuations. Thus, we shall assume the existence of multiple 'aims' of the multiple criteria of the programming problem. Let these be represented in the form,

$$(5.3) \qquad \Lambda(x, z) = (\Lambda_1(x, z), \ldots, \Lambda_i(x, z), \ldots, \Lambda_n(x, z))$$

where Λ_i corresponds to the i^{th} criteria.

If the values of z were assumed to be known as in the case of the 'ex-post' problem, the above criteria would all be functions of a single variable x, and hence the problem is similar to the problem considered in Section 4.2.2. There we said that, in a problem with multiple 'aims', the best that could be done by the 'planning authority' was to obtain a 'Pareto optimum' set of solutions, and the optimum activity level has to be obtained in accordance of the 'welfare function' of the 'policy maker',

$$W(\Lambda_1, \Lambda_2, \cdots, \Lambda_n) .$$

We shall assume here, as in Section 4.2.2, that the explicit form of this welfare function is now known.

Now consider the 'ex-ante' problem. We shall assume that the a priori probability distribution of the random variable z is known. In decision under uncertainty, when the a priori probability distribution is known, the most appropriate proceedure is Bayes solution. The 'Pareto optimum' set of the Bayes solution can be obtained in the following manner.

Let,

$$(5.4) \qquad \Lambda_i(x) = \int_z \Lambda_i(x, z)\, p(z)\, dz \quad \text{for} \quad i = 1, 2, \ldots, n$$

and

$$(5.5) \qquad \Lambda(x) = \{\Lambda_1(x), \Lambda_2(x), \ldots, \Lambda_n(x)\} .$$

The 'Pareto optimum' set defined for the vector (5.5) will be the 'Pareto optimum' Bayes solution for the 'ex-ante' problem.

If the *a priori* probability distribution is not known, other criteria for decision under uncertainty such as minimax principle, minimax loss principle, minimax regret principle and minimum most likely criteria are available in the literature on decision under uncertainty. The 'Pareto optimum' solution for the 'ex-ante' problem with multiple 'aims' could be defined for each of these criteria in a manner similar to the Bayes form. Since in the problems that we will be considering, the *a priori* probability distribution is assumed to be known, we would be concerned with only the Bayes form of the solution.

5.3 Allocation within the Export Sector.

In this section we shall consider the problem of allocation within the export sector of the economy, as a problem in programming under uncertainty. The export sector; as we saw in Chapter 2, produced commodities purely for export. The prices for the products were assumed to be determined in the world market and were subject to fluctuations. The output of these commodities too, depending on climatic conditions, and hence were subject to fluctuations. It was further assumed that the whole of the product was sold in the world market at the going price. We shall now assume that there are r possible commodities that could be produced in this sector of the economy, each of these commodities having a character similar to the aggregate commodity considered in Chapter 3. That is, each of these commodities has a production function, price mechanism and returns similar to the aggregate commodity, with the values of the coefficient differing among the different commodities. We shall assume that the distribution in total proceeds (output x price) for each of the commodities is a random variable of known probability distribution.

We also assume that there exist two maximising criteria for investment policy in this sector of the economy. (1) The maximisation of the total returns from the sector. (2) The minimisation of the fluctuation in the income of this sector.

The programming problem under uncertainty can be formally stated as follows:

Let the output of the i^{th} of these commodities be of the form,

$$X_{1i} = f_i(L_i)\, z_i \qquad i = 1, 2, \text{---}\, r \tag{5.6}$$

where $f_i(L_i)$ is the production function of the i^{th} industry, having diminishing returns to land and having a form similar to equation (3.1) of Chapter 3.

The decision variable is the amount of capital that is to be allocated, capital to land ratio is assumed to be constant,

$$L_i = \kappa_{1i}\, K_{1i} \qquad i = 1, 2, \dots , r \tag{5.7}$$

where κ_{1i} are assumed to be constants.

Land is assumed to be available in unrestricted amounts, but the returns to each additional unit of land diminish with each additional unit of land, and it is the total available capital that is the limiting resource of allocation. The variable representing strategy of 'state and nature' is the random variable z.

We have the following programming problem;

$$\text{(5.8)} \qquad \max. \quad \Lambda_{11} = E[\sum_{i=1}^{r} f(L_i)\, z_i]$$

$$\text{(5.9)} \qquad \max. \quad \Lambda_{12} = -V = -F' \,\Omega\, F$$

$$\text{(5.10)} \qquad \text{subject to} \quad \sum \kappa_{1i} \leq \bar{K}_1, \qquad f_i(L_i) \geq 0$$

where

$$\text{(5.11)} \qquad F = \begin{pmatrix} f_1(L_1) \\ f_2(L_2) \\ \\ f_r(L_r) \end{pmatrix}$$

and Ω is a variance-covariance matrix; w_{ij} the ij^{th} term, which corresponds to the covariance of $z_i z_j$.

There is one major difference between the above formulation and that in Chapter 3. In Chapter 3, we showed that profit maximisation lead to the working of the industry under full capacity for low enough values of z. Here, for the simplicity of analysis, we shall assume that the industries of this sector will work at full capacity for all values of z.

The problem is one of multiple criteria, the 'best' that the 'planning authority' could do is, therefore, the determination of the 'Pareto optimum set'. The choice of the optimum activity vector from the

'Pareto optimum' set will be made by the 'policy maker' according to the Social welfare function, which is not known to the 'planning authority'.

First we shall determine the 'Pareto optimum' set of activities in the allocation problem with two industries and then later extend to the case of many industries.

Let

$$x_i = \frac{X_{1i}}{z_i} = f_i(L_i) \ , \tag{5.12}$$

and then

$$K_{1i} = \frac{1}{\kappa_{1i}} L_i = \frac{1}{\kappa_{1i}} f_i^{-1} (x_i) \tag{5.13}$$

The problem of allocation between two industries of the export sector can be stated as,

$$\text{max.} \quad \Lambda_{11} = x_1 \mu_1 + x_2 \mu_2 \tag{5.8'}$$

$$\text{max.} \quad \Lambda_{12} = -V = - [x_1^2 \sigma_1^2 + 2x_1 x_2 \sigma_{12} + x_2^2 \sigma_2^2] \tag{5.9'}$$

$$\text{subject to,} \quad \frac{1}{\kappa_{11}} f_1^{-1}(x_1) + \frac{1}{\kappa_{12}} f^{-1}(x_2 \leq \bar{K} \tag{5.10'}$$

$$x_i \geq 0$$

where

$$\mu_i = E(z_i) \tag{5.14}$$

and

$$(5.15) \qquad \sigma_i^2 = \text{Var } z_i \;, \quad \text{and} \quad \sigma_{12} = \text{Cov } z_1 z_2$$

Now, for each value of Λ_{11}, we shall determine the value of x that maximises Λ_{12}. Let,

$$(5.16) \qquad \psi = x_1^2 \sigma_1^2 + 2x_1 x_2 \sigma_{12} + x_2^2 \sigma_2^2 + 2\lambda[\Lambda_{11} - \mu_1 x_1 - \mu_2 x_2]$$

Therefore,

$$(5.17) \qquad \begin{aligned} \frac{\partial\psi}{\partial x_1} &= 2x_1 \sigma_1^2 + 2x_1 \sigma_{12} - 2\lambda \mu_1 \\ \frac{\partial\psi}{\partial x_2} &= 2x_1 \sigma_{12} + 2x_2 \sigma_2^2 - 2\lambda \mu_2 \end{aligned}$$

and $$\Lambda_{11} = \mu_1 x_1 + \mu_2 x_2$$

$x' = [x_1', x_2']$ the solution to equations (5.17), represents the maximum of Λ_{12} for a given value of Λ_{11}, i.e., x' is a 'Pareto optimum' point. Thus, the locus of all such points would represent the 'Pareto optimum' set for the above problem, without the capital constraint (5.10'). If x' is a point of this locus, it satisfies the relation,

$$(5.18) \qquad \lambda = \frac{1}{\mu_1}[x_1 \sigma_1^2 + x_2 \sigma_{12}] = \frac{1}{\mu_2}[x_1 \sigma_{12} + x_2 \sigma_2^2]$$

i.e., the equation of the unconstrainted 'Pareto optimum' is given by,

$$(5.19) \qquad \frac{\sigma_1^2 \, x_1' + \sigma_{12} \, x_2'}{\sigma_{12} \, x_1' + \sigma_2^2 \, x_2'} = \frac{\mu_1}{\mu_2}$$

or

$$(5.20) \qquad \frac{x_1'}{\mu_1 \, \sigma_2^2 - \sigma_{12} \, \mu_2} = \frac{x_2'}{\mu_2 \, \sigma_1^2 \, \mu \, \sigma_{12} \, \mu_1}$$

Therefore, the 'Pareto optimum' set of unconstrained problem can be represented by the positive ray defined by the relation,

$$(5.21) \qquad \frac{x_1}{A} = \frac{x_2}{B} = \rho$$

where

$$(5.22) \qquad \begin{aligned} A &= \mu_2 \, \sigma_2^2 - \sigma_{12} \, \mu_1 \\ B &= \mu_1 \, \sigma_1^2 - \sigma_{12} \, \mu_2 \end{aligned}$$

and x_1, x_2 are positive and feasible. ρ could be considered as a distance parameter.

Equation (5.10') represents the capital restriction, this is an non-linear relation in x_1 and x_2. We shall now determine the nature of this restriction for a simple form of the production function of the export sector. Let the production functions take the following simple form,

$$(5.23) \qquad x_1 = f_1(L_1) = \beta_{11} \, L_1 - \beta_{12} \, L_1^2$$

(5.24) $$x_2 = f_2(L_2) = \beta_{21} L_2 - \beta_{22} L_2^2$$

where the β's are constants. It is clear that if β_{12} and β_{22} are positive the assumption of diminishing returns will be satisfied.

Eliminating L_2 between (5.24) and the equality represented by (5.10') expressed in the form,

$$\frac{1}{\kappa_{11}} L_1 + \frac{1}{\kappa_{12}} L_2 = \bar{K}$$

we get,

(5.25)
$$\begin{aligned} x_2 &= \beta_{21} \kappa_{12}(\bar{K} - \frac{1}{\kappa_{11}} L_1) - \beta_{22} \kappa_{12}^2(\bar{K} - \frac{1}{\kappa_{11}} L_1)^2 \\ &= (\beta_{21} \kappa_{12} \bar{K} - \beta_{22} \kappa_{12}^2 \bar{K}^2) + (\beta_{21} \frac{\kappa_{12}}{\kappa_{11}} + \frac{2\beta_{22} \kappa_{12}^2 \bar{K}}{\kappa_{11}})L_1 \\ &\qquad - \beta_{22} \frac{\kappa_{12}^2}{\kappa_{11}^2} L_1^2 \\ &= \beta_o + \beta_1 L_1 - \beta_2 L_1^2 \end{aligned}$$

Now eliminating L_1 between (5.25) and (5.23) we get,

(5.26) $$\frac{L_1^2}{\begin{vmatrix} -\beta_{11} & x_1 \\ -\beta_1 & x_2 - \beta_o \end{vmatrix}} = \frac{L_1}{\begin{vmatrix} x_1 & \beta_{12} \\ x_2 - \beta_o & \beta_2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} \beta_{12} & \beta_{11} \\ \beta_2 & \beta_1 \end{vmatrix}}$$

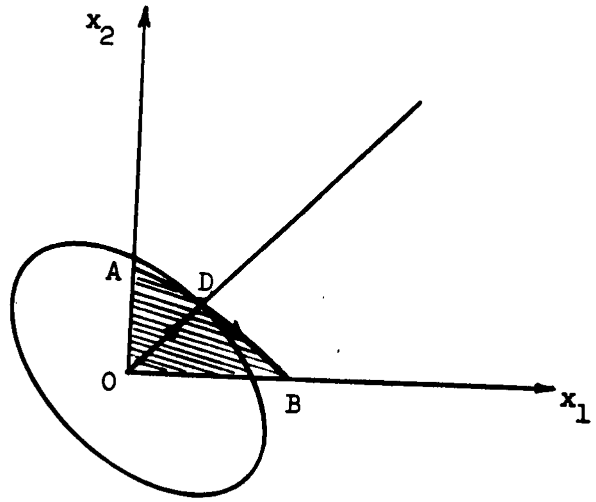

Figure 1

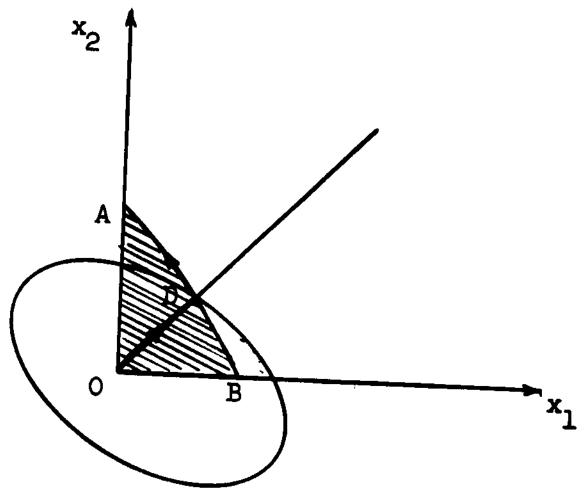

Figure 2

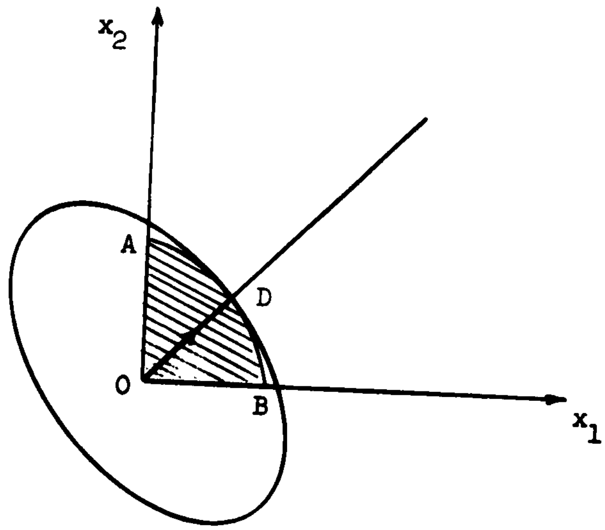

Figure 3

Therefore, the capital restriction or the production possibility curve for maximum output, for the two industries can be represented in the form,

$$(5.27) \quad [x_1 \beta_2 - \beta_{12}(x_2 - \beta_2)]^2 = (\beta_1 \beta_{12} - \beta_2 \beta_{11})[\beta_1 x_1 - \beta_{11}(x_2 - \beta_o)] .$$

This is an equation of a parabola.

In the x_1, x_2 space the first 'aim' Λ_1 could be represented by parallel straight lines, and Λ_2 by equi-variance ellipses. The ray given by the equation (5.21) corresponds to the locus of tangents points of Λ_2 to Λ_1.

In Figures 1, 2, and 3 let AB represent the capital restriction. Then the feasible set of activities is given by the area OAB. Also let D be the point at which (5.21) intersects the capital restriction barrier AB. To every point on the interior of OAB there is a point on OD or on AB, which has the same value for Λ_1 but a higher value for Λ_2, and hence preferable to it. It is then clear that OD constitutes a part of the 'Pareto optimum' set. The rest of the 'Pareto optimum set, as we shall see, will be a part of the curve AB.

Three possibilities, as represented by the above three figures could arise depending on the manner in which the equi-variance curve intersects AB at D.

In Figure 1, the slope of the equi-variance curve at D is greater than that of AB. From the figure it is clear that the 'Pareto optimum' set in this case is given by ODB. But if the slopes are reversed as in Figure 2, the 'Pareto optimum' set is given by ODA. The third possibility is when the two curves touch at D, as in Figure 3.

In this case the 'Pareto optimum' set would consist of only the part of the ray OD. The other possibility AB touching the equi-variance curve from outside, does not change this result.

The optimum activity vector could be determined from the 'Pareto optimum' set if the 'welfare function' of the 'policy maker' were known.

Consider the following simple form of the 'welfare function' of the 'policy maker',

$$W(\Lambda_1 \ \Lambda_2) = \Lambda_1 + \alpha \Lambda_2 \tag{5.28}$$

If we substitute for Λ_1 and Λ_2, we see that this equation is a quadratic function in x_1 and x_2. If we substitute for x_1 and x_2 from equation (5.21), (5.28) becomes a function of the single variable ρ; further, this is an increasing function ρ. Therefore, the behavior of this function can be discussed as a function of the single variable ρ. The special interest of the above case is that, any quadratic form can be expressed in a form similar to (5.28), and thus the method of solution of the above problem can be extended to the solution of problems of quadratic programming.[6/]

We shall now extend the solution of the above two industry cases to the general r industry problem.

[6/] The method of solution of quadratic programming problems discussed by Markovitz, in the journal article and the book cited above, is very similar to this method.

The programming problem for the general case can be written as,

$$(5.8'') \qquad \text{max.} \qquad \Lambda_{11} = \sum_{i=1}^{r} \mu_i \, x_i = \mu' \, x$$

$$(5.9'') \qquad \text{max.} \qquad \Lambda_{12} = -V = -\, x' \, \Omega \, x$$

$$(5.10'') \qquad \text{subject to} \qquad \sum_{i=1}^{r} \frac{1}{\kappa_{1i}} f'(x_i) \leq \bar{K}$$

and $\qquad x_i \geq 0$

$$\text{where} \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_r \end{pmatrix} \quad \text{and} \quad \mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_r \end{pmatrix}$$

and $\qquad \mu_i = E(z_i)$

and Ω is the variance covariance matrix of $z_1, z_2, \ldots, z_r$.

The 'Pareto optimum' set for the unrestricted programming problem can be in the following form. Let

$$(5.29) \qquad \psi = x' \, \Omega \, x + 2\lambda[\Lambda_{11} - \mu' \, x]$$

differenciating,

$$(5.30) \qquad \frac{\partial \psi}{\partial x} = 2x' \, \Omega - 2\lambda \, \mu$$

thus, the maximum value of Λ_{12} for any given value of Λ_{11}, satisfies the equation,

$$(5.31) \qquad x = \lambda \, \Omega^{-1} \, \mu$$

Therefore, we can represent the 'Pareto optimum' of the unrestricted case by the following equation,

$$(5.32) \qquad \frac{x_1}{\Omega_1\ \mu} = \frac{x_2}{\Omega_2\ \mu} = \cdots = \frac{x_i}{\Omega_i\ \mu} = \cdots \frac{x_r}{\Omega_r\ \mu} = \rho$$

where Ω_i is a vector representing the i^{th} row of the matrix Ω^{-1}, and ρ is a distance parameter.

The equation (5.32) defines the equation of a ray through the origin; for, positive values of x this would be a ray in the positive quadrant.

The determination of the explicit form of the capital restriction (5.10"), which is also the maximum production possibility surface for the sector, is rather difficult. However, because of the assumptions of diminishing returns to the different industries of this sector, it can be proved that this surface is concave. Let this surface be expressed in the form,

$$(5.33) \qquad Q(\bar{K}) = Q(x_1, \ \ldots \ , x_r)$$

Then the 'Pareto optimum' set for the above problem including the capital restriction can be determined by maximising the function ψ with the added restriction (5.33). i.e., consider the lagrangian,

$$(5.34) \qquad \psi' = x' \ \Omega \ x + 2\lambda_1(\Lambda_{11} - \mu' \ x) + 2\lambda_2[Q(\bar{K}) - Q(x)]$$

differenciating we get the equations,

$$(5.35) \qquad 2\,\Omega\, x - 2\lambda_1\, \mu - 2\lambda_2 \frac{\partial Q(x)}{\partial x} = 0$$

where $\frac{\partial Q}{\partial x}$ is a vector $\left\{ \frac{\partial Q(x)}{\partial x_1} \cdots \frac{\partial Q(x)}{\partial x_i} \cdots \frac{\partial Q}{\partial x_r} \right\}$

Thus, the 'Pareto optimum' set for the above general problem would consist of the part of the ray (5.32) lying in the feasible set, and a curve on the surface (5.33) given by the equations (5.35).

CHAPTER 6

ALLOCATION OF RESOURCES WITHIN SECTORS: (ii) DOMESTIC MANUFACTURING SECTOR

6.1 Introduction.

In this final chapter we shall take up the problem of allocation of resources within the domestic manufacturing sector of the economy. We have seen that the demand for the products of this sector are subject to random variations. This is different from the problem considered in the previous chapter, where the random variations arose in the prices and the output.

The analysis of this problem too shall be considered as a problem in programming under uncertainty. However, the basic form of this programming problem is different from the programming problem that we considered in the previous chapter. There the random variables occurred in the criteria function, while here they arise in the restriction in that the industries would only meet existing demand.

The problem under consideration is the determination of the optimum capacity of a set of industries when the demand for their products are uncertain. The capacity being determined before knowing the demand, this is again an 'ex-ante' problem. Similar problems in programming under uncertainty have been discussed by Danzig[1/] and Elmagraby.[2/] The problem that we shall be considering however, is of a more general nature, and we believe, has a possibility of wider applicability, than to the solution of the immediate problem.

1/ 'Linear Programming Under Uncertainty', op. cit.

2/ 'Programming Under Uncertainty', op. cit.

In the next section, therefore, we shall discuss this programming under uncertainty problem in a more general context. And, in the following section apply these results to the specific problem, allocation within the domestic manufacturing sector of our model.

6.2 Inter-industry Model with Random Demand.

6.2.1 The Programming Problem and its Maximising Criteria. The problem is that of allocation of resources among a group of r industries, when the demand for the products of these industries are uncertain.

The random variables in this programming problem arise in the restrictions. The output is assumed to be less than or equal to demand, and this demand is a random variable. As discussed in Section 5.2 of Chapter 5, when the random variables arise in the restrictions, we have the possibility of non-feasible solutions of the programming problem. In discussing the methods of solution of such problems, we concluded that the most appropriate proceedure for their solution was to associate the losses arising from non-feasibility into the maximising criteria.

Further, in the present problem, we shall assume only a single 'aim' or criteria; the maximisation of the total net returns from the group of industries. Thus, we would not have to obtain a 'Pareto optimum' as we did in the previous chapter.

Let d_j be the demand for the product of the j^{th} industry, and let the cumulative probability distribution of d_j given by $F_j(d_j)$, $d_j \geq 0$.

It is assumed that the output of each industry in any given period of time, is only to meet the demand for that period. Thus, when demand is less than output at full capacity, capacity would be less than fully utilised.

Let the returns at full capacity of the j^{th} industry be X_j, and let v_j represent demand inexcess (if any) of maximum output, and s_j the extent of under utilisation of capacity. i.e.,

$$(6.1) \qquad v_j = d_j - X_j \qquad d_j > X_j$$
$$= 0 \qquad \text{Otherwise}$$

and

$$(6.2) \qquad s_j = X_j - d_j \qquad d_j < X_j$$
$$= 0 \qquad \text{Otherwise}$$

Also,

$$(6.3) \qquad d_j = X_j + v_j - s_j$$

and

$$v_j > 0 \qquad s_j = 0$$

$$s_j > 0 \qquad v_j = 0$$

The last two restrictions can also be written in the form,

$$v_j \, s_j = 0 \qquad \text{with} \qquad v_j \geq 0 \quad s_j \geq 0$$

It is assumed that excess demand results in losses of a known form, and that these could be measured in the same units, in which the returns to the industry are measured. Such losses could arise for instance due to the commodities having to be imported, causing a drain

on foreign exchange. Let the loss function due to excess demand be represented in the form,

$$\xi_j(v_j)\ , \qquad v_j \geq 0$$

Similarly there would be losses arising due less than full utilisation of capacity. Let this function be represented in the form,

$$\eta_j(s)\ , \qquad s \geq 0$$

We make the assumptions,

$$\xi_j(0) = \eta_j(0) = 0$$

and that $-\xi_j(x)$ and $-\eta_j(x)$ are concave functions.

Thus the net gain to the j^{th} industry can be represented in the form,

$$(6.4) \qquad \begin{aligned} \varphi_j(X_j|d_j) &= X_j - \xi_j(v_j) \qquad && d_j > X_j \\ &= d_j - \eta_j(s_j) && d_j < X_j \end{aligned}$$

The total gain from the r industries can be represented in the form,

$$(6.5) \qquad \varphi(X_1,\ \ldots\ ,\ x_r|d_1,\ \ldots\ ,\ d_r) = \sum_{j=1}^{r} \varphi_j(X_j|d_j)$$

This function, which incorporates the losses due to non-feasibility, is the new maximisation criteria of the programming problem. We see

that this function depends on the random variables, $d_1\ d_2\ \cdots\ d_r$. This problem in decision under uncertainty is similar to that which arose in the previous chapter. We concluded in that connection that if the probability distribution of the random variables were known, the 'best' available proceedure was the maximisation of the expected value, we shall do the same here.

The total expected gain from the r industries is,

$$(6.6)\qquad \begin{aligned} \Phi = E(\varphi) &= E[\sum_{j=1}^{r} \varphi_j(X_j|d_j)] \\ &= \sum_{j=1}^{r} \{E[\varphi_j(X_j|d_j)]\} \end{aligned}$$

Therefore, we see that the total expected gain is equal to the sum of the expected gains of the different industries.

The expected gain of the j^{th} industry can be obtained from equation (6.4) in the form,

$$(6.7)\qquad \begin{aligned} E(\varphi_j) = X_j &- \int_0^{X_j} \{(X_j - d_j) + \eta_j(X_j - d_j)\}\, d\, F_j(d_j) \\ &- \int_{X_j}^{\infty} \xi_j(d_j - X_j)\, d\, F_j(d_j) \end{aligned}$$

Since the functions $-\xi_j(x)$ and $-\eta_j(x)$ are assumed to be concave, it follows from the theorem stated in Chapter 4, (page 97), that $E(\varphi_j)$ is a concave function in X_j.

The maximum value of this concave function is given by the equation,

$$
(6.8)\qquad
\begin{aligned}
\frac{d}{dX_j} E(\varphi_j) &= 1 - \eta_j(0)\, p_j(X_j) + \xi_j(0)\, p_j(X_j) \\
&\quad - \int_0^{X_j} \{1 + \eta_j'(X_j - d_j)\}\, dF_j + \int_{X_j}^{\infty} \xi_j'(d_j - X_j)\, dF_j \\
&= 1 - \int_0^{X_j} \{1 + \eta_j'(X_j - d_j)\}\, dF_j + \int_{X_j}^{\infty} \xi_j'(d_j - X_j)\, dF_j \\
&= 0
\end{aligned}
$$

where $p_j(X_j) = \frac{d}{dX_j} F_j(d_j)$, and ξ_j' and η_j' denotes the first derivatives of $\xi_j(x)$ and $\eta_j(x)$ with respect to x.

We next evaluate the above function for the simple case when the loss functions are linear, i.e.,

$$
(6.9)\qquad
\begin{aligned}
\xi_j(x) &= \xi_j \cdot x \\
\eta_j(x) &= \eta_j \cdot x
\end{aligned}
$$

where ξ_j and η_j are now constants.

We have,

$$
(6.10)\qquad
\begin{aligned}
E(\varphi_j) &= X_j - (1 + \eta_j) \int_0^{X_j} (X_j - d_j)\, dF_j \\
&\qquad - \xi_j \int_{X_j}^{\infty} (d_j - X_j)\, dF_j \\
&= X_j - (1 + \eta_j) \cdot [X_j\, F_j(X_j) - \delta(X_j)] \\
&\qquad - \xi_j[\bar{d}_j - \delta(X_j)]
\end{aligned}
$$

where

$$(6.11) \qquad \delta(X_j) = \int_0^{X_j} d_j \, dF_j(d_j)$$

$$(6.12) \qquad \delta'(X_j) = \int_{X_j}^{\infty} d_j \, dF_j(d_j)$$

and, the mean of the demand of the j^{th} industry,

$$(6.13) \qquad E(d_j) = \bar{d} = \delta(X_j) + \delta'(X_j)$$

Equation (6.8) reduces to the form,

$$(6.14) \qquad \begin{aligned} \frac{d}{dX_j} [E(\varphi_j)] &= 1 - (1 + \eta_j) F_j(X_j) + \xi_j [1 - F_j(X_j)] \\ &= 1 - (1 + \xi_j + \eta_j) F_j(X_j) + \xi_j \end{aligned}$$

Thus the unconstrained maximum of the criteria function is given by,

$$(6.15) \qquad F_j(X_j) = \frac{1 + \xi_j}{1 + \eta_j + \xi_j} \leq 1 \qquad j = 1, 2, \ldots, r.$$

Given the probability distributions of d_j, the valued of that satisfies this equation can be obtained, let these values be represented in the form,

$$(6.16) \qquad X_j = F^{-1}\left(\frac{1 + \xi_j}{1 + \xi_j + \eta_j} \right), \quad j = 1, 2, \ldots, r.$$

These represent the optimum capacity for the unconstrained problem.

We note further that,

$$\frac{d^2}{dX^2}[E(\varphi_j)] = -(\eta_j + \xi_j)\, f(X_j) \leq 0 \tag{6.17}$$

which also shows the concavity of (6.7).

6.2.2 Input-output Model. Thus far we have not specified the production function of the different industries. We shall now assume that the r industries form an input-output model.

The r products, therefore, are assumed to be used in the production of other commodities. Let X_{ij} be the input of the i^{th} commodity used in the production of the j^{th} commodity, and X_{io} output of the i^{th} commodity produced for final demand.

The production function of the j^{th} industry is assumed to be of the form,

$$X_j = \min \left\{ \frac{X_{1j}}{a_{1j}}, \frac{X_{2j}}{a_{2j}}, \ldots, \frac{X_{rj}}{a_{rj}}, \frac{X_{r+1,j}}{a_{r+1,j}} \right\} \tag{6.18}$$

where X_j is the total output of the j^{th} commodity, and a_{ij} constants denoting the minimal input of the i^{th} industry, per unit output of the j^{th} commodity.

Then we have, i.e.,

$$\begin{aligned} X_i &= \sum_{j=1}^{r} X_{ij} + X_{io} \\ &= \sum a_{ij} X_j + X_{io} \qquad i = 1, 2, \ldots, r \end{aligned} \tag{6.19}$$

These equations can be represented in the form,

$$(6.20) \qquad [I - A]\, X = X_o$$

where,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1r} \\ \vdots & a_{ij} & \\ a_{1r} & \cdots & arr \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdot & & \vdots \\ \vdots & & & \ddots & 0 \\ 0 & & \cdots & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ \vdots \\ X_r \end{bmatrix}, \quad X_o = \begin{bmatrix} X_{10} \\ \vdots \\ X_{ro} \end{bmatrix}$$

Therefore,

$$(6.21) \qquad X = [I - A]^{-1} X_o = \mathcal{A}\, X_o$$

where,

$$(6.22) \qquad \mathcal{A} = [1 - A]^{-1} = \begin{bmatrix} A_{11} & \cdots & A_{1r} \\ \vdots & A_{ij} & \vdots \\ A_{r1} & \vdots & A_{rr} \end{bmatrix}$$

Equation (6.22) gives us the relation between the output for final demand and the total output of these commodities.

We further assume that there is one constraint. Arising from a factor which is not produced within the model, let the input of this factor onto the j^{th} industry be $a_{r+1,j}$ and the total endowment of this factor be $\bar{a}_{r+1}$.

Then we have,

$$(6.23) \qquad \sum_{j=1}^{r} a_{r+1,j}\, X_j \leq \bar{a}_{r+1}$$

or

$$a'_{r+1} X \leq \bar{a}_{r+1} \qquad \text{where} \qquad a'_{r+1} = [a_{r+1,1}, \cdots, a_{r+1,r}]$$

From equation (6.21) we get,

$$a'_{r+1} \mathcal{A} X_o \leq \bar{a}_{r+1} \tag{6.24}$$

let,

$$a'_{r+1} \mathcal{A} X_o = \sum_{i=1}^{r} A_{r+1,i} X_{io} \tag{6.25}$$

where,

$$A_{r+1,i} = \sum_{j=1}^{r} a_{r+1,j} A_{ji} \tag{6.26}$$

The programming problem under consideration for linear loss functions can be represented in the form,

$$\text{max.} \qquad \Phi = \sum_{j=1}^{r} E(\varphi_j)$$

$$\text{subject to,} \quad X = \mathcal{A} X_o$$

$$\text{and} \qquad \sum_{i=1}^{r} A_{r+1,i} X_{io} \leq \bar{a}_{r+1}$$

The solution to this problem can be considered in the following manner:

(a) First consider the problem without the factor restriction.

The optimum output for final demand in this case is given as in equation (6.16), in the form

$$(6.16) \qquad X_{jo} = F^{-1}\left\{\frac{1 + \xi_j}{1 + \xi_j + \eta_j}\right\}, \quad j = 1, 2, \ldots, r .$$

The equivalent total output is then given as,

$$X_i = \sum_{j=1}^{r} A_{ij}\, X_{jo} = \sum_{j=o}^{r} A_{ij}\; F^{-1}\left\{\frac{1 + \xi_j}{1 + \xi_j + \eta_j}\right\}, \quad i = 1, 2, \ldots, r$$

If this solution is feasible (or if the factor constraint is not binding), i.e., if

$$\sum_{j=1}^{r} a_{r+1,j}\, X_j < \bar{a}_{r+1}$$

this is the optimum solution to the problem.

(b) If the above solution is not feasible, then the factor constraint is binding. Then consider the lagrangian,

$$(6.27) \qquad \Phi' = \sum_{j=1}^{r} E[\varphi_j(X_{jo})] + \lambda[\bar{a}_{r+1} - \sum_{j=1}^{r} A_{r+1,j}\, X_{jo}]$$

Differenciation and using the result of equation (6.14) we get

$$(6.28) \qquad \frac{\partial \Phi'}{\partial X_{jo}} = [1 - (1 + \eta_j + \xi_j)\, F_j(X_{jo}) + \xi_j] - \lambda\, A_{r+1,j}$$

$$j = 1, 2, \ldots, r$$

which gives us,

$$(6.29) \qquad F_j(X_{jo}) = \frac{1 + \xi_j - \lambda A_{r+1,j}}{1 + \eta_j + \xi_j} \qquad \lambda A_{r+1,j} < 1 + \xi_j$$

$$= 0 \qquad \lambda A_{r+1,j} > 1 + \xi_j$$

also,

$$[1 - (1 + \eta_j + \xi_j)\ F_j(X_{jo}) + \xi_j]\ X_{jo} = \lambda A_{r+1,j}\ X_{jo}$$

Taking summation over j and using equation (6.25) we get,

$$(6.30) \qquad \lambda = \frac{1}{a_{r+1}} \sum_{j=1}^{r} [1 - (1 + \eta_j + \xi_j)\ F_j(X_{jo}) + \xi_j]\ X_{jo}$$

Equations (6.29) and (6.30) defines values optimum valued for output for final demand, from equation (6.21) the total output can be determined.

The above analysis could be easily extended to the general case of m factor restrictions.

6.3 Allocation Within the Domestic Manufacturing Sector.

We shall now consider the problem of allocation of resources within the domestic manufacturing sector of the economy, as an application of the results derived in Section 6.2.

The demand for the aggregate commodity of the domestic manufacturing sector was derived in equation (3.48) to be of the form,

$$d_2 = b_{20} + b_{21}\ L + b_{22}\ f(L)\ z$$

We shall now assume that the demand is for each of the commodities of the domestic manufacturing sector to behave in a manner similar to the aggregate commodity. Therefore, let the demand for the j^{th} product of this sector be of the same form,

$$(6.31) \qquad d_{2j} = b_{20j} + b_{21j} L + b_{22j} f(L) z \qquad j = 1, 2, \dots, r$$

where the b's are constants. As before L corresponds to the land under cultivation of the subsistence sector, and z the random variable arising from the fluctuations in the prices and output of the export sector.

Also, let the returns of the j^{th} industry, in a manner similar to the aggregate commodity in equation (3.64), be expressed as,

$$(6.32) \qquad \begin{aligned} X_{2j} &= \bar{X}_{2j} ; && z > \bar{\bar{z}} \\ &= X^*_{2j} , && \bar{z} < z < \bar{\bar{z}} ; \quad j = 1, 2 — r \\ &= X^{**}_{2j} , && z < \bar{z} \end{aligned}$$

where $\bar{\bar{z}}_j$ (as $\bar{\bar{z}}$ in Chapter 3) denotes the value of z for which

$$(6.33) \qquad d_{2j} = \bar{X}_{2j} \qquad j = 1, 2 — r$$

and $\bar{z}$ as before is the value of z at which the export sector reaches full capacity, as such it is independent of X_{2j}, and therefore it is the same for all j.

In the production function of the aggregate commodity of this sector we assumed constant capital output and constant capital labor ratios. We shall make similar assumptions for each of the industries of this sector.

We shall further assume that as in the case of the aggregate commodity of this sector, that labor is available in unrestricted quantities and as such is not a restricting factor of production, and the only factor of production is capital, let the capital available for allocation in this sector be $\bar{K}_2$.

For the rest of this analysis, we shall for simplicity drop the first suffix 2 from these coefficients (denoting that the coefficients correspond to the domestic manufacturing sector). Since we are only dealing with coefficients in the domestic manufacturing sector, for the rest of this chapter this will not cause any ambiguity.

The 'aim' of this sector is assumed to be the maximisation of the net returns to this sector. Thus, assuming linear loss functions, as given by equation (6.9), for under utilisation of capacity and demand in excess of full capacity production, we obtain the following expected net gain function for the j^{th} industry, in a manner similar to equation (6.10) as,

$$
\begin{aligned}
E(\varphi_j) = \bar{X}_j &- (1+\eta_j)\int_0^{\bar{z}} (\bar{X}_j - X_j^*)\, p_j(z)\, dz \\
&- (1+\eta_j)\int_{\bar{z}}^{\bar{\bar{z}}} (\bar{X}_j - X_j^{**})\, p_j(z)\, dz \qquad (6.34) \\
&- \xi_j \int_{\bar{\bar{z}}}^{\infty} (\bar{X}_j - d_j)\, p(z)\, dz
\end{aligned}
$$

This is a concave function of X_j, and its maximum value is given by,

$$0 = \frac{d}{dX_j} E(\varphi_j) = 1 - (1 + \eta_j)\int_0^{\bar{z}} p_j(z)\, dz - (1 + \eta_j)\int_{\bar{z}}^{\bar{\bar{z}}_j} p_j(z)\, dz$$

$$(6.35) \qquad + \xi_j \int_{\bar{\bar{z}}_j}^{\infty} p_j(z)\, dz$$

$$= 1 - (1 + \eta_j)\, F_j(\bar{\bar{z}}) + \xi_j[1 - F_j(\bar{\bar{z}})]$$

$$= 1 + \xi_j - (1 + \xi_j + \eta_j)\, F(\bar{\bar{z}})$$

Thus the maximum return from the unconstrained problem is obtained when,

$$F(\bar{\bar{z}}_j) = \frac{1 + \xi_j}{1 + \xi_j + \eta_j} \leq 1$$

i.e.,

$$(3.36) \qquad \bar{\bar{z}}_j = F^{-1}\left[\frac{1 + \xi_j}{1 + \xi_j + \eta_j}\right] \qquad \text{for} \quad j = 1, 2, \text{---} r$$

Now this value of $\bar{\bar{z}}_j$ defines the optimum capacity from equation (6.33).

We shall as in Section 6.2 now include into the production function of these industries the products of the r industries themselves.

Thus, let X_{ij} be the input of the i^{th} commodity in the production of the j^{th} commodity. And if a_{ij} denotes the constant input coefficients, and X_{jo} the amount of the j^{th} commodity produced for final demand, we get as in equation (6.21),

$$X = \mathcal{A}\, X_o$$

where X and X_o denote, as before, the vectors of total output X_j and output for final demand X_{jo} respectively, and $\mathcal{A}$ is given by equation (6.22).

If $a_{r+1,j}$ is the constant capital coefficient of the j^{th} industry we have,

$$\sum_{j=1}^{r} a_{r+1,j} X_j \leq \bar{K}_2 \tag{6.37}$$

As before this can be written in the form,

$$\sum_{j=1}^{r} A_{r+1,j} X_{jo} \leq \bar{K}_2 \tag{6.38}$$

Thus the problem of allocation of resources between the r industries of the export sector can be expressed as the following programming problem.

$$\text{max.} \qquad \Phi = \sum_{j=1}^{r} \{E[\varphi_j(X_{jo})]\}$$

$$\text{subject to} \qquad X = \mathcal{A} X_o$$

$$\text{and} \qquad \sum_{j=1}^{r} A_{r+1,j} X_{jo} \leq \bar{K}$$

The solution to this problem, as in Section 6.2, can be considered in the following manner:

(a) First consider the solution to the problem without the capital constraint (6.37).

The solution to this problem was given by equation (6.36) as

$$\bar{\bar{z}}_j = F^{-1}\left[\frac{1 + \xi_j}{1 + \xi_j + \eta_j}\right] \qquad j = 1, 2, \ldots, r$$

which equation now defines the optimum values of output for final demand. From equation (6.22) we obtain the optimum values of the total production of these industries.

If this solution is feasible, then this is the 'optimum' solution.

(b) If the above solution is not feasible, then it follows that the capital constraint is binding.

In this case considering the lagrangian function,

$$\Phi' = \sum_{j=1}^{r} E[\varphi_j(X_{jo})] + \lambda[\bar{K} - \sum_{j=1}^{r} A_{r+1,j}\, X_{jo}] \tag{6.39}$$

we get,

$$\begin{aligned} \frac{\partial\Phi}{\delta X_{jo}} &= [1 - (1 + \quad{}_j + \xi_j)\, F_j(\bar{\bar{z}}_j) + \xi_j] - \lambda\, A_{r+1,j} \\ &= 0 \qquad\qquad j = 1, 2, \ldots, r \end{aligned} \tag{6.40}$$

i.e.,

$$\begin{aligned} F_j(\bar{\bar{z}}_j) &= \frac{1 + \xi_j - \lambda\, A_{r+1,j}}{1 + \eta_j + \xi_j} \qquad \text{for} \quad \lambda A_{r+1,j} < 1 + \xi_j \\ &= 0 \qquad\qquad \text{for} \quad \lambda A_{r+1,j} > 1 + \xi_j \end{aligned} \tag{6.41}$$

and λ is given by the relation,

$$\lambda = \frac{1}{\bar{K}} \sum_{j=1}^{r} [1 - (1 + \eta_j + \xi_j) F_j(\bar{\bar{z}}_j) + \xi_j] X_{jo} \tag{6.42}$$

Equations (6.41) and (6.42) define values for $\bar{\bar{z}}_j$. These values of $\bar{\bar{z}}$ define the 'optimum' output X_{jo} that need be produced for final demand. Finally these X_{jo} define the output at full capacity from equation (6.22), and hence we get the 'optimum' capacity levels of the r industries.

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